

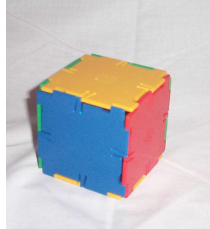
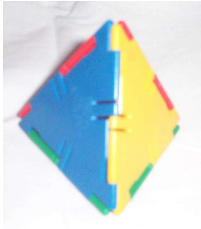
Name _____

Group: Blue Green Yellow

Summer Educational Enrichment in Math, 2022

Math Contest

1. **Platonic Solids:** Name the 5 Platonic Solids and say how many faces they have. (Spelling does not matter.)



Name _____

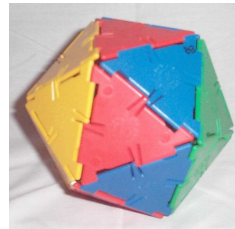
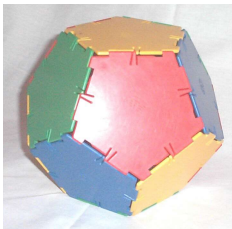
Name _____

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Faces _____

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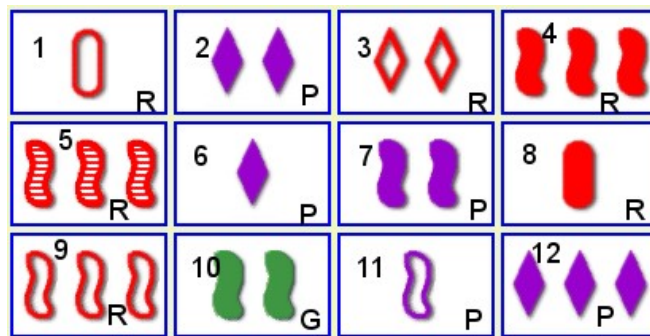
Name _____

Name _____

Faces _____

Faces _____

2. **Sets:** Find as many sets as possible from the cards shown below. Recall: a set consists of 3 cards whose characteristics (color, shape, number, shading) are all the same or all different. Use the numbers in each square to form your sets (e.g. 1, 2, 3). The letters refer to the color of the card (G=green, P=purple, and R=red). There are **fewer** than 8 sets!



Set 1:

Set 2:

Set 3:

Set 4:

Set 5:

Set 6:

Set 7:

Set 8:

3. **Euler numbers:** Consider the octahexahedron made from 6 squares and 8 triangles:



The number of faces is:

The number of vertices is: Explain below.

The number of edges is: Explain below.

Explain V : (Don't use the Euler number.)

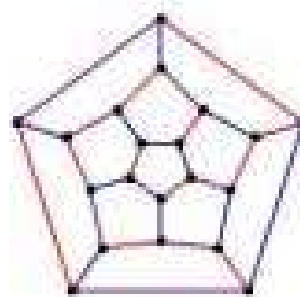
Explain E : (Don't use the Euler number.)

Calculate the Euler number:

Explain how you know the Euler number before counting F , V and E ?

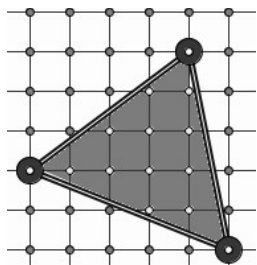
4. **Map Coloring:** The map at the right has 11 countries. Color it with as few colors as possible. Countries with a common edge must have different colors. Use the abbreviations:

R =red B =blue G =green Y =yellow P =purple



Explain why you cannot do it with fewer colors.

5. **Pick's Theorem:** For the shaded region below, state the number of internal points and the number of boundary points, and find the area using Pick's Theorem:



6. Counting Cubes:

- a. The perfect squares are the sequence: 1, 4, 9, 16, 25, 36, ...
 - i. List the first 5 differencing numbers for the perfect squares.

 - ii. What is another name for the differencing numbers for the perfect squares?

- b. The perfect cubes are the sequence: 1, 8, 27, 64, 125, 216, ...
 - i. List the first 5 differencing numbers for the perfect cubes.

 - ii. What is another name for the differencing numbers for the perfect cubes?

7. Rational Tangles: You have two ropes which were tangled using Twists (T) and Rotations (R). The tangle is assigned the rational number $\frac{-4}{3}$. Write down the list of Twists and Rotations which will untangle the ropes and the rational number assigned to each intermediate step. (There may be more blanks than you need.)

T/R _____

$\frac{-4}{3} \Rightarrow$ _____ \Rightarrow _____ \Rightarrow _____ \Rightarrow _____ \Rightarrow _____ \Rightarrow _____ \Rightarrow _____

T/R _____

\Rightarrow _____ \Rightarrow _____ \Rightarrow _____ \Rightarrow _____ \Rightarrow _____ \Rightarrow _____ \Rightarrow _____

8. Euler Circuits: Here is a map of the 5 bridges crossing the San Francisco Bay connecting 3 land masses.



- a. Is it possible to travel so that you cross all 5 bridges without crossing any bridge twice?

Yes No

- b. Is it possible to travel so that you cross all 5 bridges without crossing any bridge twice and you end up where you started?

Yes No

- c. Draw your route. Put an *A* where you start and a *B* where you finish.

9. **Logic Problem:** A very special island is inhabited only by knights and knaves. Knights always tell the truth, and knaves always lie.

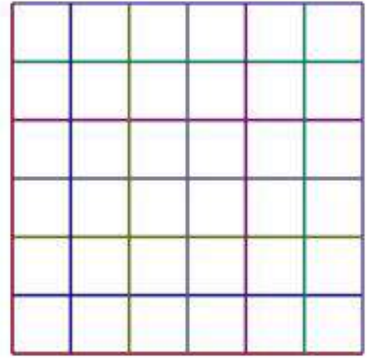
You meet two inhabitants: Betty and Peggy. Betty tells you "Peggy is a knave." Peggy tells you, "Betty and I are both knights." Can you determine who is a knight and who is a knave? Explain your reasoning.

10. **Probability:**

- a. **Birthday Problem:** What is the probability that Polly and Jason have different birthdays (assuming neither was born in a leap year)?
- b. **Birthday Problem:** If 5 people are in a room, what is the probability that at least 2 of them have birthdays in the same month?
- c. **Strings:** You hold 4 strings in your hand. You tie off 2 pairs at the top and 2 pairs at the bottom. When you let go, what is the probability that the strings are all in one loop?

11. **Pigeonhole Principle:** There are 49 students at SEE-Math this year. Is it true that at least 5 of you are born in the same month? Explain why or why not.

12. **Counting Diagonals:** What is the largest number of diagonals that can be drawn in a 6×6 grid with no 2 touching? $D_6 = \underline{\hspace{2cm}}$
 Draw them.



13. **Magic Cards:**
- Convert 46 base 10 to base 3.
 - Convert 34 base 5 to base 2.

14. **Bulgarian Solitaire:** In a Bulgarian Solitaire game, the initial partition is (4,2,1). What are the next 5 partitions? What is the period of the final loop?

$(4,2,1) \Rightarrow (\quad) \Rightarrow (\quad) \Rightarrow (\quad) \Rightarrow (\quad) \Rightarrow (\quad)$

Period = $\underline{\hspace{2cm}}$

15. **Gerrymandering:** A class with 25 students will be divided into 5 groups and each group will elect 1 student to the prom committee. Of the 25 students, 10 will vote for the NERDS (X) and 15 will vote for the JOCKS (O). Is it possible for the nerds to elect more representatives to the prom committee? If yes, show how by putting 5 X's and O's in each box below.

Circle one: Yes No

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16. **Pop-Tac-Toe:** It is Blue's turn.
 Can Blue win on this turn?
 If so, which square should Blue play on?
 If there is more than one answer,
 just list one winning play.

Yes No

Play _____



A8	B8	C8	D8	E8	F8	G8	H8
A7	B7	C7	D7	●	F7	G7	H7
A6	B6	C6	D6	E6	F6	G6	H6
●	B5	●	D5	●	F5	●	H5
A4	B4	C4	D4	E4	●	G4	H4
A3	B3	●	D3	E3	F3	G3	H3
A2	B2	C2	D2	E2	F2	G2	H2
A1	B1	C1	D1	●	F1	G1	H1



17. **Kenken:** Solve the Kenken:

1-	2-		10×	
	9+	9+		
4		2-		
6+		2÷		10+
	2÷			

18. **Cryptography:** Solve the cryptogram:

Q L W V W B T Q S M O Z M M V M O O A I V L P I U.

Q L W V W B T Q S M B P M U, A I U - Q - I U.

Hint: I → A