MATH 151, FALL 2011
COMMON EXAM II - VERSION B

Last Name: *KEY* **First Name**: ______________

Signature: ______________ **Section No**: ______________

PART I: Multiple Choice (15 questions, 4 points each. No Calculators)
Write your name, section number, and version letter (B) of the exam on the ScanTron form.

1. Find the derivative of $g(x) = \dfrac{x^3+1}{x^2+1}$.

(a) $g'(x) = \dfrac{x^4+3x^2-2x}{(x^2+1)^2}$ Correct choice

(b) $g'(x) = \dfrac{5x^4+3x^2+2x}{x^2+1}$

(c) $g'(x) = \dfrac{x^4+3x^2+2x}{(x^2+1)^2}$

(d) $g'(x) = \dfrac{x^4+3x^2-2x}{x^2+1}$

(e) $g'(x) = \dfrac{5x^4+3x^2+2x}{(x^2+1)^2}$

2. A ball is thrown vertically upward with a velocity of 80 feet per second and the height, s, of the ball at time t seconds is given by $s(t) = 80t - 16t^2$. What is the velocity of the ball when it is 96 feet above the ground on its way up?

(a) 112 ft/sec

(b) 24 ft/sec

(c) 16 ft/sec Correct choice

(d) 48 ft/sec

(e) 64 ft/sec

3. Which of the following vectors is tangent to the curve $\mathbf{r(t)} = \left\langle \sqrt{t^2+1}, t \right\rangle$ at the point $(2, \sqrt{3})$?

(a) $\left\langle \dfrac{1}{\sqrt{5}}, 1 \right\rangle$

(b) $\left\langle \dfrac{1}{2}, 1 \right\rangle$

(c) $\left\langle \dfrac{\sqrt{3}}{4}, 1 \right\rangle$

(d) $\left\langle \dfrac{2}{\sqrt{5}}, 1 \right\rangle$

(e) $\left\langle \dfrac{\sqrt{3}}{2}, 1 \right\rangle$ Correct choice

4. Find the 81^{st} derivative of $f(x) = \dfrac{1}{x}$.

(a) $f^{(81)}(x) = -\dfrac{(81)!}{x^{81}}$

(b) $f^{(81)}(x) = \dfrac{(80)!}{x^{80}}$

(c) $f^{(81)}(x) = -\dfrac{(81)!}{x^{82}}$ Correct choice

(d) $f^{(81)}(x) = -\dfrac{(80)!}{x^{80}}$

(e) $f^{(81)}(x) = \dfrac{(81)!}{x^{81}}$

5. $\lim\limits_{x \to -\infty} (9 - 7e^{-x}) =$

(a) $-\infty$ Correct choice

(b) 0

(c) ∞

(d) 7

(e) 9

6. At what point on the graph of $f(x) = \sqrt{x}$ is the tangent line parallel to the line $2x - 3y = 4$?

(a) $\left(\dfrac{16}{9}, \dfrac{4}{3}\right)$

(b) $\left(\dfrac{9}{16}, 0\right)$

(c) $\left(\dfrac{4}{3}, \dfrac{2}{\sqrt{3}}\right)$

(d) $\left(\dfrac{9}{16}, \dfrac{3}{4}\right)$ Correct choice

(e) $\left(\dfrac{1}{16}, \dfrac{1}{4}\right)$

7. Given the equation $2xy + \pi \sin(y) = 2\pi$, find $\dfrac{dy}{dx}$ when $x = 1$ and $y = \dfrac{\pi}{2}$.

(a) $-\dfrac{\pi}{2 - \pi}$

(b) $-\dfrac{\pi}{3}$

(c) $-\dfrac{\pi}{2 + \pi}$

(d) 0

(e) $-\dfrac{\pi}{2}$ Correct choice

8. Find the equation of the tangent line to the graph of $f(x) = \dfrac{x}{1+2x}$ at $x = 1$.

(a) $y - \dfrac{1}{3} = -\dfrac{1}{9}(x-1)$

(b) $y - \dfrac{1}{3} = -\dfrac{4}{9}(x-1)$

(c) $y - \dfrac{1}{3} = \dfrac{x}{1+2x}(x-1)$

(d) $y - \dfrac{1}{3} = \dfrac{1}{9}(x-1)$ Correct choice

(e) $y - \dfrac{1}{3} = -\dfrac{x}{1+2x}(x-1)$

9. If $f(x) = \sin(g(x))$, find $f'(2)$ given that $g(2) = \dfrac{\pi}{3}$ and $g'(2) = \dfrac{\pi}{4}$.

(a) $\dfrac{\pi}{8}$ Correct choice

(b) $\dfrac{\sqrt{3}\pi}{8}$

(c) $\dfrac{1}{2}$

(d) $-\dfrac{\sqrt{3}\pi}{8}$

(e) $-\dfrac{\pi}{8}$

10. $\displaystyle\lim_{x\to 0} \frac{\sin^3(4x)}{x^3} =$

(a) ∞

(b) 64 Correct choice

(c) 1

(d) 0

(e) 4

11. Find the slope of the tangent line to the curve $x = t^2 + t + 1$, $y = \sqrt{t} + 4$ at $t = 9$.

(a) $\dfrac{1}{114}$ Correct choice

(b) $\dfrac{3}{5}$

(c) $\dfrac{19}{6}$

(d) $\dfrac{5}{12}$

(e) 114

12. Find the derivative of $h(t) = (t^4 + 7t)^5$.

(a) $h'(t) = 5(4t^3 + 7)^4$
(b) $h'(t) = 5(t^4 + 7t)^4(4t^3)$
(c) $h'(t) = 5(t^4 + 7t)(4t^3 + 7)$
(d) $h'(t) = 20t^{19} + 7^5(5t^4)$
(e) $h'(t) = 5(t^4 + 7t)^4(4t^3 + 7)$ Correct choice

13. Given $f(x)$ is a one-to-one function, find $g'(3)$ where g is the inverse of the function $f(x) = x^9 + x^3 + x$.

(a) $g'(3) = \dfrac{1}{12}$
(b) $g'(3) = 1$
(c) $g'(3) = \dfrac{1}{13}$ Correct choice
(d) $g'(3) = \dfrac{1}{9}$
(e) $g'(3) = 13$

14. Find the derivative of $f(x) = x^3 e^{2x}$.

(a) $f'(x) = 3x^2 e^{2x} + 2x^3 e^{2x}$ Correct choice
(b) $f'(x) = 6x^2 e^{2x}$
(c) $f'(x) = 3x^2 e^{2x} + x^3 e^{2x}$
(d) $f'(x) = 3x^2 e^{2x}$
(e) $f'(x) = 3x^2 e^{2x} + 2x^4 e^{2x-1}$

15. Find the linear approximation, $L(x)$, for $f(x) = \sqrt[3]{x}$ at $x = -8$.

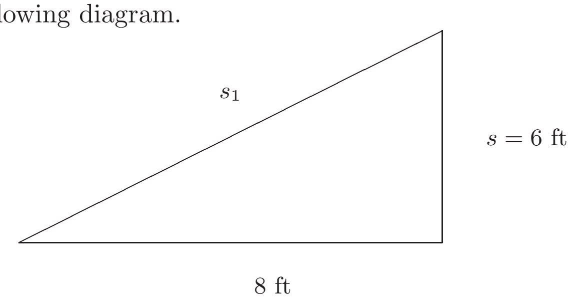
(a) $L(x) = -2 + \dfrac{1}{12}(x + 8)$ Correct choice
(b) $L(x) = -2 - \dfrac{1}{12}(x + 8)$
(c) $L(x) = -2 + \dfrac{1}{12}(x - 8)$
(d) $L(x) = -2 - \dfrac{1}{12}(x - 8)$
(e) $L(x) = -2 + \dfrac{1}{4}(x + 8)$

PART II WORK OUT

Directions: Present your solutions in the space provided. *Show all your work* neatly and concisely and *Box your final answer*. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

16. (8 pts) An observer is standing 8 feet from the base of a balloon launching point. At the instant the balloon has risen vertically 6 feet, the height of the balloon is increasing at a rate of 10 feet per minute. How fast is the distance from the observer to the balloon changing at this same instant? Assume the balloon starts on the ground and rises vertically.

Solution: The setup yields the following diagram.



By the Pythagorean Theorem we have

$$s_1^2 = 8^2 + s^2.$$

Differentiating with respect to t yields

$$\begin{align} 2s_1 \frac{ds_1}{dt} &= 0 + 2s\frac{ds}{dt} \tag{1} \\ \frac{ds_1}{dt} &= \frac{s}{s_1}\frac{ds}{dt}. \tag{2} \end{align}$$

When $s = 6$ feet, we have $s_1 = \sqrt{8^2+6^2} = 10$ feet (again by the Pythagorean Theorem), and we are given $\frac{ds}{dt} = 10\, ft/min$.

Substituting these values into equation (2), we obtain that the distance from the observer to the balloon is changing/increasing by

$$\frac{ds_1}{dt} = \frac{6}{10}10 = 6 \ \text{ft/min}$$

17. (8 pts) Find the second derivative of $f(x) = \tan(x^3)$.

Solution: By the chain rule we have

$$f'(x) = 3x^2 \sec^2(x^3).$$

For the second derivative we use the product rule

$$f''(x) = 6x\sec^2(x^3) + 3x^2 \frac{d}{dx}\sec^2(x^3),$$

and then twice the chain rule for the term

$$\frac{d}{dx}\sec^2(x^3) = 2\sec(x^3)[\sec(x^3)\tan(x^3)]3x^2 = 6x^2\sec^2(x^3)\tan(x^3).$$

Therefore, we obtain

$$f''(x) = 6x\sec^2(x^3) + 18x^4\sec^2(x^3)\tan(x^3).$$

18. (8 pts) A rain gauge has the shape of a cone with the vertex at the bottom whose radius is half of the height. Given that the volume of a cone is $V = \frac{1}{3}\pi r^2 h$, find the differential dV in terms of only h and the differential dh. Use the differential dV to estimate the change in volume when the height of water in the gauge increases from 5 cm to 5.3 cm.

<u>Solution:</u> Substituting $r = h/2$ into the formula for V, we obtain

$$V(h) = \frac{1}{12}\pi h^3 \qquad \Rightarrow \qquad dV = V'(h)dh = \frac{1}{4}\pi h^2 dh$$

and the change in volume when the height of water in the gauge increases from 5 cm to 5.3 cm can be estimated by

$$dV = \frac{1}{4}\pi(5)^2(5.3-5) = \frac{15}{8}\pi \ cm^3.$$

19. (8 pts) For the equation $y = e^{2x} + e^{-3x}$, show $y'' + y' - 6y$ is a constant. Find the constant.

<u>Solution:</u> Compute the derivatives

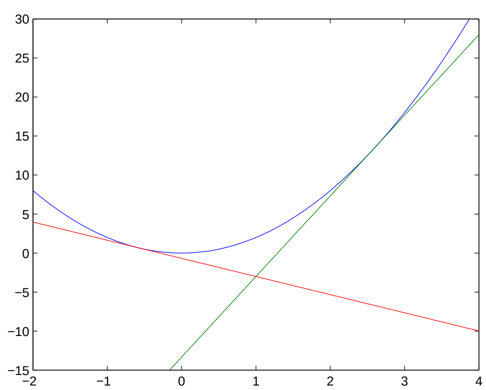
$$\begin{aligned} y' &= 2e^{2x} - 3e^{-3x} \\ y'' &= 4e^{2x} + 9e^{-3x}. \end{aligned}$$

Substitution into $y'' + y' - 6y$ yields

$$y'' + y' - 6y = 4e^{2x} + 9e^{-3x} + 2e^{2x} - 3e^{-3x} - 6(e^{2x} + e^{-3x}) = 0$$

which is a constant.

20. (8 pts) Draw a diagram to show there are two tangent lines to the parabola $y = 2x^2$ that pass through the point $(1, -3)$ by sketching the parabola and both tangent lines on the grid provided below. Find the x-coordinates where these tangent lines touch the parabola.



For $y = f(x) = 2x^2$ we have

$$f'(x) = 4x.$$

The family of tangent lines at the point $(a, f(a))$ on the parabola is given by

$$y = f(a) + f'(a)(x-a) = 2a^2 + 4a(x-a) = 4ax - 2a^2.$$

The point with the coordinates $(x, y) = (1, -3)$ is on this family of tangent line, exactly if

$$\begin{aligned} -3 &= 4a - 2a^2 \\ 2a^2 - 4a - 3 &= 0. \end{aligned}$$

Solving this quadratic equation, we obtain x-coordinates

$$x = a = 1 \pm \frac{\sqrt{10}}{2}$$

at which these tangent lines touch the parabola.