Student (Print) _____ Section _____

Last, First Middle

Student (Sign) _____

Student ID _____

Instructor _____

MATH 152
Exam1
Spring 2000
Test Form A

Part I is multiple choice. There is no partial credit.

Part II is work out. Show all your work. Partial credit will be given.

You may not use a calculator.

1-10	/50
11	/10
12	/10
13	/10
14	/10
15	/10
TOTAL	

1-10	/50
11	/10
12	/10
13	/10
14	/10
15	/10
TOTAL	

Formulas:

$$\sin(A)\sin(B) = \frac{1}{2}\cos(A-B) - \frac{1}{2}\cos(A+B)$$

$$\sin(A)\cos(B) = \frac{1}{2}\sin(A-B) + \frac{1}{2}\sin(A+B)$$

$$\cos A\cos B = \frac{1}{2}\cos(A-B) + \frac{1}{2}\cos(A+B)$$

$$S_n = \frac{1}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)\right]\Delta x$$

$$|E_T| \le \frac{K(b-a)^3}{12n^2} \quad \text{where } K \ge |f''(x)| \text{ for } a \le x \le b$$

$$|E_S| \le \frac{K(b-a)^5}{180n^4} \quad \text{where } K \ge \left|f^{(4)}(x)\right| \text{ for } a \le x \le b$$

$$\int \sec\theta\, d\theta = \ln|\sec\theta + \tan\theta| + C$$

$$\int \csc\theta\, d\theta = \ln|\csc\theta - \cot\theta| + C$$

$$\int \ln x\, dx = x\ln x - x + C$$

Part I: Multiple Choice (5 points each)

There is no partial credit. You may not use a calculator.

1. Compute $\int_0^{1/2} xe^{2x}\,dx$.

a. $\frac{1}{2}e - \frac{1}{4}$

b. $\frac{1}{4}e^2$

c. $\frac{1}{4}(e^2 - 1)$

d. $\frac{1}{4}$

e. $\frac{3}{4}$

2. Compute $\int_0^{\pi} \cos^2(x)\,dx$.

a. 1

b. $\frac{\pi}{2}$

c. 2

d. $\frac{\pi}{2} - 1$

e. π

3. Find the average value of the function $y = xe^{x^2}$ on the interval $0 \le x \le 2$.

a. $\frac{1}{2}(e^4 - 1)$

b. $\frac{1}{4}(e^4 - 1)$

c. e

d. $\frac{1}{2}e^4$

e. e^4

4. Use the Trapezoid Rule with $n = 2$ to approximate $\int_1^5 \frac{1}{x}\,dx$.

a. $\frac{14}{15}$
b. $\frac{23}{15}$
c. $\frac{28}{15}$
d. $\frac{46}{15}$
e. $\frac{76}{45}$

5. If you use the Trapezoid Rule with $n = 8$ to estimate $\int_1^5 \frac{1}{x}\,dx$ you obtain the approximation $T_8 = 1.628968$. (Don't compute this.) Use the Trapezoid Error formula

$$|E_T| \le \frac{K(b-a)^3}{12n^2} \quad \text{where } K \ge |f''(x)| \text{ for } a \le x \le b$$

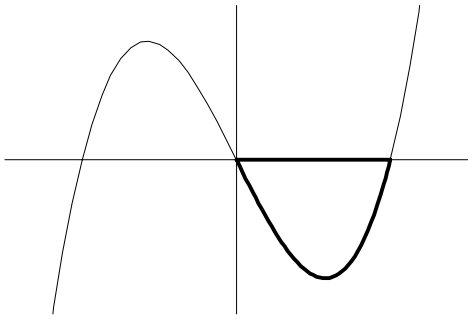
to find an upper bound for the error in this approximation.

a. $|E_T| \le \frac{1}{6}$
b. $|E_T| \le \frac{1}{60}$
c. $|E_T| \le \frac{1}{72}$
d. $|E_T| \le \frac{1}{300}$
e. $|E_T| \le \frac{1}{750}$

6. Compute $\int_0^{\pi/4} \tan^2(x)\sec^4(x)\,dx$

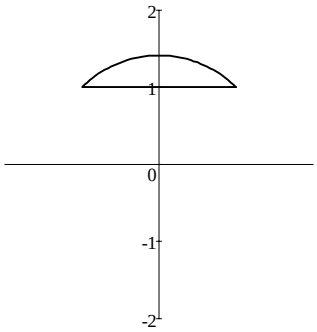
a. $\frac{\sqrt{2}}{2} - \ln\left(1 - \frac{\sqrt{2}}{2}\right)$
b. $\frac{\sqrt{2}}{2} + \ln\left(1 - \frac{\sqrt{2}}{2}\right)$
c. $\frac{8}{15}$
d. $\frac{4}{35}$
e. 4

7. Find the volume of the solid obtained by rotating the region bounded by $y = x^3 - x$ and $y = 0$ for $x \geq 0$ about the y-axis.



a. $\frac{4}{15}\pi$
b. $\frac{1}{2}\pi$
c. $\frac{2}{15}\pi$
d. $\frac{8}{105}\pi$
e. $\frac{16}{105}\pi$

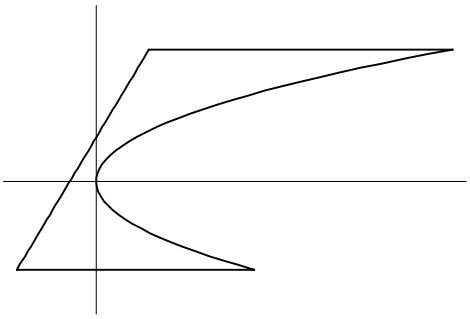
8. Which integral gives the volume of the solid obtained by rotating the region bounded by $x^2 + y^2 = 2$ and $y = 1$ about the x-axis?



a. $\int_{-1}^{1} \pi\left((2 - x^2)^2 - 1\right) dx$
b. $\int_{-\sqrt{2}}^{\sqrt{2}} \pi\left(\sqrt{2 - x^2} - 1\right)^2 dx$
c. $\int_{0}^{1} 2\pi x(1 - x^2)\, dx$
d. $\int_{0}^{1} 2\pi x\left(\sqrt{2 - x^2} - 1\right) dx$
e. $\int_{-1}^{1} \pi(1 - x^2)\, dx$

9. Find the area of the region bounded by

$x = 3y^2$, $x - 2y = -2$, $y = -2$ and $y = 3$.



a. 8
b. 24
c. 40
d. 56
e. 64

10. Which of the following definite integrals equals $\int_1^3 \sqrt{x^2 - 2x + 5}\, dx$?

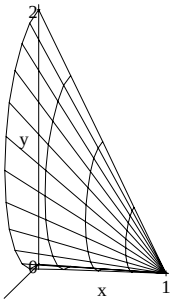
a. $4\int_0^{\pi/2} \sin\theta\, d\theta$
b. $2\int_0^{\pi/2} \sin^2\theta\, d\theta$
c. $2\int_0^{\pi/4} \tan\theta\, d\theta$
d. $4\int_0^{\pi/4} \sec\theta\, d\theta$
e. $4\int_0^{\pi/4} \sec^3\theta\, d\theta$

Part II: Work Out (10 points each)

Show all your work. Partial credit will be given.

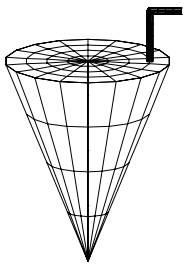
You may not use a calculator.

11. An object has a base that is the triangle with vertices $(0,0)$, $(1,0)$, and $(0,2)$. The crosssections perpendicular to the x-axis are semicircles. Find the volume of the object. You must use an integral.



12. Compute $\int \frac{1}{x^2\sqrt{x^2-4}}\,dx$.

13. A tank has the shape of a circular cone with height 20 m, radius 8 m and vertex down. It has a spout which extends 5 m above the top of the tank. The tank is initially full of water. Set up the integral that gives the work required to pump all the water out of the spout. Do not evaluate the integral.



water density $= 1000 \frac{\text{kg}}{\text{m}^3}$ $\quad g = 9.8 \frac{\text{m}}{\text{sec}^2}$

14. Compute $\int_1^e x \ln x \, dx$.

15.

a. Find the partial fraction expansion for $\frac{x-2}{x^3+x}$.

b. Then compute $\int \frac{x-2}{x^3+x}\,dx$.