Student (Print) ______________________________ Section ________

Last, First Middle

Student (Sign) ______________________________

Student ID ______________________________

Instructor ______________________________

MATH 152

Exam 2

Spring 2000

Test Form A

1-10	/50
11	/10
12	/10
13	/10
14	/10
15	/10
TOTAL	

Part I is multiple choice. There is no partial credit.

Part II is work out. Show all your work. Partial credit will be given.

You may not use a calculator.

Part I: Multiple Choice (5 points each)

There is no partial credit. You may not use a calculator.

1. Compute $\int_{-1}^{1} \frac{1}{x^6}\, dx$.

- **a.** 0
- **b.** $\frac{2}{5}$
- **c.** $-\frac{2}{7}$
- **d.** $-\frac{2}{5}$
- **e.** Divergent

2. Find the solution to $\frac{dy}{dx} = -\frac{x}{y}$ satisfying the initial condition $y(3) = 4$. Then $y(0) =$

- **a.** 0
- **b.** 1
- **c.** 3
- **d.** 5
- **e.** 6

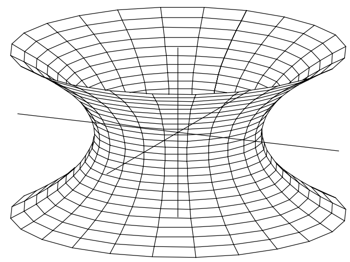
3. A tank contains 500 liters of water with 10 kg of sugar dissolved. Sugar water that contains $\frac{1}{10}$ kg of sugar per liter of water flows into the tank at the rate of 7 liters per minute. Sugar water that contains $\frac{1}{20}$ kg of sugar per liter of water flows into the tank at the rate of 8 liters per minute. The solution is kept thoroughly mixed and drains from the tank at the rate of 15 liters per minute. Write down the initial value problem for $S(t)$, the amount of sugar in the tank at time t.

- **a.** $\frac{dS}{dt} = 10t + \frac{3}{10} - \frac{S}{500}$ with $S(0) = 100$
- **b.** $\frac{dS}{dt} = \frac{11}{10} - \frac{3S}{100}$ with $S(0) = 10$
- **c.** $\frac{dS}{dt} = \frac{3}{20} - \frac{S}{500}$ with $S(0) = 10$
- **d.** $\frac{dS}{dt} = \frac{3}{20} - \frac{3S}{100}$ with $S(0) = \frac{1}{50}$
- **e.** $\frac{dS}{dt} = \frac{11}{10}t - \frac{S}{500}$ with $S(0) = 100$

4. Which integral gives the surface area of the "spool" obtained by revolving the curve

$y^2 - x + 1 = 0$ for $-1 \leq y \leq 1$

about the y-axis.



a. $\int_{-1}^{1} 2\pi(1+y^2)\sqrt{1+4y^2}\,dy$

b. $\int_{-1}^{1} 2\pi y\sqrt{1+4y^2}\,dy$

c. $\int_{-1}^{1} \pi y\sqrt{1+2y}\,dy$

d. $\int_{-1}^{1} 2\pi(1+4y^2)\sqrt{1+y^2}\,dy$

e. $\int_{-1}^{1} 2\pi(1+y^2)\sqrt{1+2y^2}\,dy$

5. Find the x-coordinate of the centroid (center of mass, $\bar{x}$) of the region between $y = x^3$ and the x-axis for $0 \leq x \leq 2$.

a. $\frac{4}{5}$

b. 4

c. $\frac{8}{5}$

d. $\frac{32}{5}$

e. $\frac{2}{3}$

6. Find the limit of the sequence $a_n = \dfrac{\ln(n^2+1)}{\ln(n)}$.

a. 0
b. $\ln 2$
c. 2
d. e
e. ∞

7. Which of the following series are convergent?

(i) $\displaystyle\sum_{n=1}^{\infty} \frac{100^n}{n!}$ (ii) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{n+3^n}$

a. both (i) and (ii)
b. (i) only
c. (ii) only
d. neither

8. Which of the following series are convergent?

(i) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n+1}$ (ii) $\displaystyle\sum_{n=2}^{\infty} \frac{n}{\ln n}$

a. both (i) and (ii)
b. (i) only
c. (ii) only
d. neither

9. Compute $\displaystyle\sum_{n=0}^{\infty} \frac{5^{n+1}}{4^n}$.

- **a.** -20
- **b.** $\dfrac{20}{9}$
- **c.** 20
- **d.** 25
- **e.** divergent

10. The series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

- **a.** is absolutely convergent.
- **b.** is convergent but not absolutely convergent.
- **c.** is divergent to $+\infty$.
- **d.** is divergent to $-\infty$.
- **e.** is divergent but not to $\pm\infty$.

Part II: Work Out (10 points each)

Show all your work. Partial credit will be given.

You may not use a calculator.

11. Find the solution of $\frac{dy}{dx} - 3y = 6e^{-x}$ satisfying $y(0) = 5$. Solve for y.

12. Consider the series $S = \sum_{n=1}^{\infty}\left(\frac{n}{n+1} - \frac{n+1}{n+2}\right)$

a. Write out the third partial sum $S_3 = \sum_{n=1}^{3}\left(\frac{n}{n+1} - \frac{n+1}{n+2}\right)$ and add it up.

b. Compute the sum of the infinite series $S = \sum_{n=1}^{\infty}\left(\frac{n}{n+1} - \frac{n+1}{n+2}\right)$.

13. Find the arc length of the parametric curve $x = t^3$, $y = 3t^2$ between $t = 0$ and $t = \sqrt{12}$.

14. Consider the series $S = \sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$.

a. Prove the series $S = \sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$ converges. Be sure to name the convergence test you use.

b. If you approximate the series S by the partial sum $S_9 = \sum_{n=2}^{9} \frac{1}{n(\ln n)^2}$, find an upper bound for the error $|R_9| = |S - S_9|$ in the approximation and justify your estimate.

15. A triangular plate is suspended vertically in water as shown. The height of the triangle is 5 ft. Its base is 10 ft, and it is submerged 3 ft into the water. Find the force on one face of the plate.
$(\rho g = 62.5\ \frac{\text{lb}}{\text{ft}^3})$