Student (Print) ______________________________ Section __________

Last, First Middle

Student (Sign) ______________________________

Student ID ______________________________

Instructor ______________________________

MATH 152

Exam 3

Spring 2000

Test Form A

1-10	/50
11	/10
12	/15
13	/15
14	/10
TOTAL	

Part I is multiple choice. There is no partial credit.

Part II is work out. Show all your work. Partial credit will be given.

You may not use a calculator.

Part I: Multiple Choice (5 points each)

There is no partial credit. You may not use a calculator.

1. Find the values of x such that the vectors $\langle x,-1,3\rangle$ and $\langle 2,-5,x\rangle$ are orthogonal.

a. -1 only
b. 0 only
c. 1 only
d. 0 and 1 only
e. 1 and -1 only

2. Compute $\lim_{x\to 0}\dfrac{e^{x^3}-1-x^3}{x^6}$

HINT: The series for e^x may be helpful.

a. 0
b. $\dfrac{1}{3!}$
c. $-\dfrac{1}{3!}$
d. $\dfrac{1}{2}$
e. ∞

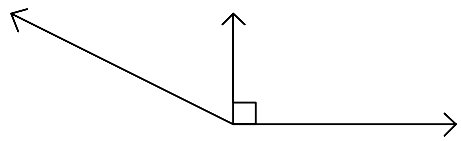
3. Consider the parametric curve $\vec{r}(t)=\langle t,\sin t,t^3\rangle$. Find parametric equations for the line tangent to the curve at $t=\pi$.

a. $x=1+\pi t,\quad y=-t,\quad z=3\pi^2+\pi^3 t$
b. $x=1+\pi t,\quad y=-1,\quad z=3\pi^2+\pi^3 t$
c. $x=\pi+t,\quad y=-1+t\cos t,\quad z=\pi^3+3t^3$
d. $x=\pi+t,\quad y=t\cos t,\quad z=\pi^3+3t^3$
e. $x=\pi+t,\quad y=-t,\quad z=\pi^3+3\pi^2 t$

4. Find the Taylor series for $f(x)=x^2+3$ about $x=2$.

a. $7+4(x-2)+(x-2)^2$
b. $7+4(x-2)+2(x-2)^2+4(x-2)^3$
c. $7+4(x-2)+(x-2)^2+\dfrac{2}{3}(x-2)^4+\cdots$
d. $7+4(x-2)+2(x-2)^2+4(x-2)^3+2(x-2)^4+\cdots$
e. $7+4(x-2)+2(x-2)^2+\dfrac{2}{3}(x-2)^3+\dfrac{4}{3}(x-2)^4$

5. The vectors $\vec{a}$, $\vec{b}$ and $\vec{c} = \vec{b} - \vec{a}$ all lie in the **same plane** as shown in the diagram. Which of the following statements is TRUE?



a. $\vec{a} \times \vec{b} = \vec{0}$.
b. $\vec{a} \times \vec{b}$ points into the page.
c. $\left(\vec{a} \times \vec{b}\right) \bullet \vec{c} = 0$.
d. $\vec{b} \times \left(\vec{a} \times \vec{c}\right)$ points in the direction of $-\vec{a}$.
e. None of These

6. Find a power series centered at $x = 0$ for the function $f(x) = \dfrac{x}{1 - 8x^3}$, and determine its radius of convergence.

a. $\displaystyle\sum_{n=0}^{\infty} (-1)^n 8^n x^{3n+1} \qquad R = \frac{1}{8}$
b. $\displaystyle\sum_{n=0}^{\infty} (-1)^n 8^n x^{3n+1} \qquad R = 8$
c. $\displaystyle\sum_{n=0}^{\infty} \frac{8^n}{n!} x^{3n+1} \qquad R = 2$
d. $\displaystyle\sum_{n=0}^{\infty} 8^n x^{3n+1} \qquad R = \frac{1}{8}$
e. $\displaystyle\sum_{n=0}^{\infty} 8^n x^{3n+1} \qquad R = \frac{1}{2}$

7. Find the distance from the point $(3, -2, 4)$ to the center of the sphere $(x - 1)^2 + (y + 1)^2 + (z - 2)^2 = 4$

a. 2
b. 3
c. 9
d. $\sqrt{61}$
e. 61

8. Let $f(x) = \sin(x^2)$. Compute $f^{(14)}(0)$, the 14^{th} derivative of $f(x)$ evaluated at 0.
HINT: Use a series for $\sin(x^2)$.

a. $-\dfrac{1}{14! \bullet 7!}$

b. $\dfrac{7!}{14!}$

c. $\dfrac{14!}{7!}$

d. $-\dfrac{14!}{7!}$

e. $-14! \bullet 7!$

9. Find the angle between the vectors $\vec{u} = \langle 1, 1, 0 \rangle$ and $\vec{v} = \langle 1, 2, 1 \rangle$.

a. $0°$

b. $30°$

c. $45°$

d. $60°$

e. $90°$

10. Evaluate the integral $\int_0^{1/2} \frac{1}{1+x^3}\, dx$ as an infinite series.

a. $\displaystyle\sum_{n=0}^{\infty} (-1)^n \left(\frac{1}{2}\right)^{3n} = 1 - \frac{1}{2^3} + \frac{1}{2^6} - \frac{1}{2^9} + \cdots$

b. $\displaystyle\sum_{n=0}^{\infty} \frac{1}{3n+1} \left(\frac{1}{2}\right)^{3n+1} = \frac{1}{2} + \frac{1}{4 \bullet 2^4} + \frac{1}{7 \bullet 2^7} + \frac{1}{10 \bullet 2^{10}} + \cdots$

c. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{3n+1} \left(\frac{1}{2}\right)^{3n+1} = \frac{1}{2} - \frac{1}{4 \bullet 2^4} + \frac{1}{7 \bullet 2^7} - \frac{1}{10 \bullet 2^{10}} + \cdots$

d. $\displaystyle\sum_{n=0}^{\infty} (-1)^n (3n-1) \left(\frac{1}{2}\right)^{3n-1} = -2 - \frac{2}{2^2} + \frac{5}{2^5} - \frac{8}{2^8} + \cdots$

e. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{3n-1} \left(\frac{1}{2}\right)^{3n-1} = -2 - \frac{1}{2 \bullet 2^2} + \frac{1}{5 \bullet 2^5} - \frac{1}{8 \bullet 2^8} + \cdots$

Part II: Work Out (points indicated below)

Show all your work. Partial credit will be given.

You may not use a calculator.

11. (10 points) Consider the planes

$$P_1 : \quad 2x - y + z = 1$$
$$P_2 : \quad x + y - 3z = 2$$

a. (2 pts) Fill in the blanks:

A normal to the plane P_1 is $\overrightarrow{N_1} =$ ______________________

A normal to the plane P_2 is $\overrightarrow{N_2} =$ ______________________

b. (3 pts) Find a vector parallel to the line of intersection of the two planes.

c. (3 pts) Find a point on the line of intersection of the two planes.

d. (2 pts) Find parametric equations for the line of intersection of the two planes.

12. (15 points) Let $f(x) = \ln x$.

a. (10 pts) Find the 3^{rd} degree Taylor polynomial T_3 for $f(x)$ about $x = 2$.

b. (5 pts) If this polynomial T_3 is used to approximate $f(x)$ on the interval $1 \le x \le 3$, estimate the maximum error $|R_3|$ in this approximation using Taylor's Inequality.

$$|R_n(x)| < \frac{M}{(n+1)!}|x-2|^{n+1} \quad \text{where } M \ge \left|f^{(n+1)}(x)\right| \text{ for } 1 \le x \le 3.$$

13. (15 points) Consider the points

$$P = (1, 0, -1), \quad Q = (2, 3, 1) \quad \text{and} \quad R = (0, 4, 1)$$

a. (5 pts) Find a vector orthogonal to the plane determined by P, Q and R.

b. (5 pts) Find the area of the triangle with vertices P, Q and R.

c. (5 pts) Find the equation of the plane determined by P, Q and R.

14. (10 points) Find the radius of convergence and the interval of convergence of the series

$$\sum_{n=0}^{\infty} \frac{1}{3^n \sqrt{n+1}} (x-2)^n.$$