Name (Print)_____ ID_____

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MATH 152 FINAL EXAM Spring 2000

Sections 513,514 P. Yasskin

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13	/13
14	/13
15	/13
16	/13

Multiple Choice: (4 points each)

1. Compute $\int_0^{\pi/2} x\cos(3x)\,dx$

a. $-\frac{\pi}{3}-\frac{1}{3}$
b. $-\frac{\pi}{6}-\frac{1}{3}$
c. $-\frac{\pi}{3}+\frac{1}{3}$
d. $-\frac{\pi}{6}-\frac{1}{9}$
e. $-\frac{\pi}{3}+\frac{1}{9}$

2. Compute $\lim_{n\to\infty}\frac{2^n}{1+3^n}$

a. 0
b. $\frac{1}{2}$
c. $\frac{1}{1-\frac{2}{3}}$
d. $\frac{\frac{1}{2}}{1-\frac{2}{3}}$
e. ∞

3. Compute $\int_0^{\pi/2}\sin^3\theta\,d\theta$

a. $-\frac{2}{3}$
b. $-\frac{1}{3}$
c. 0
d. $\frac{1}{3}$
e. $\frac{2}{3}$

4. Which formula will give the arclength of the curve $y = \sin x$ between $x = 0$ and $x = \pi$?

a. $L = \int_0^{\pi} 2\pi x\sqrt{1 + \cos^2 x}\, dx$

b. $L = \int_0^{\pi} \sqrt{1 + \cos^2 x}\, dx$

c. $L = \int_0^{\pi} 2\pi \sin x\sqrt{1 + \cos^2 x}\, dx$

d. $L = \int_0^{\pi} 2\pi x\sqrt{1 + \sin^2 x}\, dx$

e. $L = \int_0^{\pi} \sqrt{1 + \sin^2 x}\, dx$

5. Which initial value problem describes the solution to the following problem:

A 100 gal tank is initially filled with sugar water whose concentration is $0.05\frac{\text{lb sugar}}{\text{gal water}}$. Sugar is added to the tank at the rate of $2\frac{\text{lb}}{\text{hr}}$ and pure water is added at the rate of $3\frac{\text{gal}}{\text{hr}}$. The mixture is kept well mixed and drained at the rate of $3\frac{\text{gal}}{\text{hr}}$. Find the find the amount of sugar in the tank after *t* hours.

a. $\frac{dS}{dt} = 2 - 0.03S, \quad S(0) = 5$

b. $\frac{dS}{dt} = 0.1 - 0.15S, \quad S(0) = 5$

c. $\frac{dS}{dt} = 3S - 0.02, \quad S(0) = 0.05$

d. $\frac{dS}{dt} = 0.02 - 3S, \quad S(0) = 5$

e. $\frac{dS}{dt} = 0.02 - 0.03S, \quad S(0) = 0.05$

6. Find the solution of the differential equation $\frac{dy}{dx} = 2x(1 + y^2)$ satisfying the initial condition $y(2) = 0$.

a. $y = \tan(x^2) + 2$

b. $y = \tan^2(x - 2)$

c. $y = \tan(x^2 - 4)$

d. $y = \tan(x^2 + \arctan 2)$

e. $y = \tan^2(x) - \tan^2 2$

7. Compute $\int_1^2 \frac{1}{(x-2)^{2/3}}\,dx$

a. $-\infty$
b. -3
c. -1
d. 3
e. ∞

8. Compute $\lim_{x\to 0} \frac{\sin(2x) - 2x}{3x^3}$

a. $-\frac{1}{9}$
b. -4
c. $-\frac{4}{9}$
d. $-\frac{8}{9}$
e. $-\frac{4}{3}$

9. Find the radius of convergence of the series $\sum_{n=1}^{\infty} \frac{2^n}{(n+1)^2}(x-3)^n$.

a. 0
b. $\frac{1}{2}$
c. 2
d. $\frac{1}{3}$
e. 3

10. Which term is incorrect in the following partial fraction expansion?

$$\frac{-10x^2+5x^3-8x+1}{(x-1)(x-3)^2(x^2+2)} = \underbrace{\frac{A}{x-1}}_{\text{a.}} + \underbrace{\frac{B}{x-3}}_{\text{b.}} + \underbrace{\frac{D}{(x-3)^2}}_{\text{c.}} + \underbrace{\frac{Ex+F}{x^2+2}}_{\text{d.}}$$

e. They are all correct.

11. A vector $\vec{u}$ has length 3. A vector $\vec{v}$ has length 4. The angle between them is $60°$. Find $\vec{u} \bullet \vec{v}$.

a. 6
b. $\frac{1}{24}$
c. $\frac{\sqrt{3}}{24}$
d. 24
e. $6\sqrt{3}$

12. Find an equation for the plane containing the two lines

$$L_1: \quad x = 3+3t \quad y = 1+4t \quad z = 2+5t$$
$$L_2: \quad x = 3+t \quad y = 1 \quad z = 2-t$$

a. $-4x-8y-4z = 10$
b. $-4x+8y-4z = 10$
c. $x-2y+z = 3$
d. $x+2y+z = 7$
e. $x+2y+z = 10$

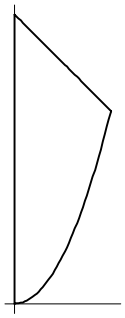
Work Out (13 points each)

Show all your work. Partial credit will be given. You may not use a calculator.

13. Compute $\int \frac{\sqrt{x^2-1}}{x}\,dx$

14. The parametric curve given by $x = t^2, \quad y = \frac{2}{3}t^3, \quad z = \frac{1}{4}t^4$ for $0 \le t \le 2$ is rotated about the y-axis. Find the area of the surface of revolution.
HINT: Factor the quantity in the square root.

15. The region in the first quadrant between the curves $y = x^2$ and $y = 6 - x$ is rotated about the y-axis. Find the volume of the solid of revolution.



16. A water tank has the shape of a circular cylinder laying on its side. It is 3 ft in radius and 5 ft long. It is half full of water. How much work is needed to pump the water out a spout at the top? (The weight density of water is $\rho g = 64.5 \frac{\text{lb}}{\text{ft}^3}$ but you may leave your answer as a multiple of ρg.)