MATH 152

Exam 3

Fall 1997

Version A

Student (Print) _____

Last, First Middle

Student (Sign) _____

Student ID _____

Instructor _____

Section _____

1-11	
12	
13	
14	
15	
TOTAL	

Part I is multiple choice. There is no partial credit. You may not use a calculator.

Part II is work out. Show all your work. Partial credit will be given. You may use your calculator.

Part I: Multiple Choice (5 points each)

There is no partial credit. You may not use a calculator. You have 1 hour.

1. If $f(x,y,z) = x^2 y \sin z$, then $\frac{\partial^2 f}{\partial x \partial z} =$

a. $\frac{xy}{z}\sin z$

b. $x^2 \sin z$

c. $2xy\sin z + x^2 y\cos z$

d. $2x^3 y^2 \sin z \cos z$

e. $2xy\cos z$

2. Which pair of vectors is NOT perpendicular?

a. $\boldsymbol{a} = \langle 1,2,3\rangle$ and $\boldsymbol{b} = \langle 3,0,-1\rangle$

b. $\boldsymbol{p} = 2\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}$ and $\boldsymbol{q} = -\mathbf{i} + \mathbf{j} + \mathbf{k}$

c. $\boldsymbol{A} = -3\mathbf{i} + 2\mathbf{j}$ and $\boldsymbol{B} = 4\mathbf{i} + 6\mathbf{j}$

d. $\boldsymbol{F} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$ and $\boldsymbol{G} = \mathbf{i} + \mathbf{j} + \mathbf{k}$

e. $\boldsymbol{u} = \langle 3,4\rangle$ and $\boldsymbol{v} = \langle 8,-6\rangle$

3. A parallelepiped has adjacent edges $\boldsymbol{u} = \langle 2,-1,4\rangle$, $\boldsymbol{v} = \langle 1,-3,0\rangle$ and $\boldsymbol{w} = \langle 3,1,-2\rangle$. Find its volume.

a. $\langle 12,4,-8\rangle$

b. -12

c. 12

d. 50

e. 54

4. The radius of a cylindrical tin can is $5\,\text{cm}$ and the height is $10\,\text{cm}$. The sides are $.01\,\text{cm}$ thick while the top and bottom are $.02\,\text{cm}$ thick each. Estimate the volume of metal used to make the can.

a. $.004\pi\,\text{cm}^3$

b. $.01\pi\,\text{cm}^3$

c. $.02\pi\,\text{cm}^3$

d. $2\pi\,\text{cm}^3$

e. $250\pi\,\text{cm}^3$

5. Which line is perpendicular to the plane $3x + 4y + 5z = 6$?

a. $\frac{x-3}{2} = \frac{y-4}{3} = \frac{z-5}{4}$

b. $x = 3 + 2t,\ y = 4 + 3t,\ z = 5 + 4t$

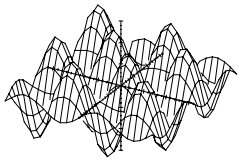
c. $x = 2 + 3t,\ y = 3 + 4t,\ z = 4 + 5t$

d. $\frac{x-2}{20} = \frac{y-3}{15} = \frac{z-4}{12}$

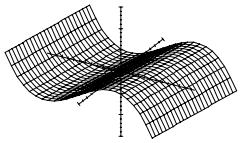
e. $x = 2 + 20t,\ y = 3 + 15t,\ z = 4 + 12t$

6. Which of the following is the graph of $f = y^2 - x^2$?

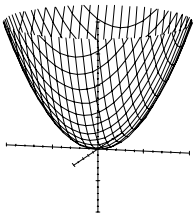
a.



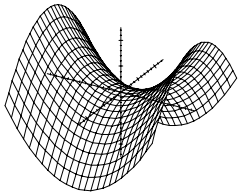
d.



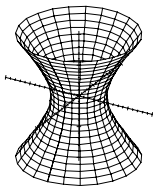
b.



e.



c.



7. Find the intersection of the line $x = 3 + 2t,\ y = 2 + t,\ z = 1 - t$ and the plane $x - y + 2z = 4$.

a. $(3, 1, 1)$

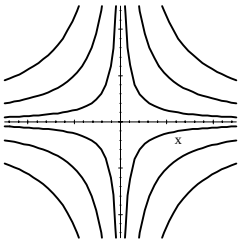
b. $(1, 1, 2)$

c. $(2, -2, 0)$

d. $(-1, 1, 3)$

e. $(0, 2, 3)$

8. For which function are the level curves (or contour plot) shown at the right?



a. $f = x^2 + y^2 - 2x$
b. $f = \cos x \cos y$
c. $f = xy$
d. $f = (x + y)^2$
e. $f = (x - y)^2$

9. An object moves in the xy-plane along the curve $y = x^2$ from $(-2, 4)$ to $(2, 4)$. In what direction does the (principal) normal $\boldsymbol{N}$ point when the object is at $(0, 0)$?

a. **j**
b. **i** + **j**
c. **j** – **i**
d. –**j**
e. **i**

10. A triangle has vertices $A = (1, 1, -1)$, $B = (2, 0, -1)$ and $C = (1, -1, 1)$. Find a vector perpendicular to the plane of the triangle.

a. $\langle 1, 1, 1 \rangle$
b. $\langle -2, 2, 2 \rangle$
c. $\langle 1, -1, 1 \rangle$
d. $\langle 2, 2, -2 \rangle$
e. $\langle 2, 2, 0 \rangle$

11. A wagon is pulled a distance of $100\,\mathrm{m}$ along a horizontal path by a constant force of $50\,\mathrm{N}$ exerted along the handle which is at $30°$ above the horizontal. How much work is done?

a. $5000\,\mathrm{J}$
b. $2500\,\mathrm{J}$
c. $2500\sqrt{3}\,\mathrm{J}$
d. $\dfrac{10000}{\sqrt{3}}\,\mathrm{J}$
e. $10000\,\mathrm{J}$

Part II: Work Out

Show all your work. Partial credit will be given.
You may use your calculator but only after 1 hour.

12. (12 points) Consider the curve $\boldsymbol{r}(t) = (t, \sin(2t), \cos(2t))$.
Compute each of the following:

a. velocity

$\boldsymbol{v} =$ _____

b. speed

$|\boldsymbol{v}| =$ _____

c. arclength between $t = 1$ and $t = 3$

$L =$ _____

d. acceleration

$\boldsymbol{a} =$ _____

e. unit tangent

$\boldsymbol{T} =$ _____

f. curvature

$\kappa =$ _____

13. (11 points) Find the plane tangent to the hyperbolic paraboloid $z = 2x^2 - y^2$ at the point $(1, 2, -2)$. Then identify its z-intercept.

14. (11 points) A particle has initial position $\mathbf{r}(t) = \langle 0, 0 \rangle$ and initial velocity $\mathbf{v}(t) = \langle 1, -1 \rangle$. If its acceleration is $\mathbf{a}(t) = \langle 4\cos(2t), 12t^2 \rangle$, find its position at $t = \pi$.

15. (11 points) Find the line of intersection of the planes $x + y + z = 3$ and $3x + y - z = 1$.