MATH 152
Final Exam
Fall 1997
Version A

Student (Print) ________________________________

Last, First Middle

Student (Sign) ________________________________

Student ID ________________________________

Instructor ____P. Yasskin________________________

Section ________________________________

1-13	
14	
15	
16	
17	
TOTAL	

Part I is multiple choice. There is no partial credit. You may not use a calculator.

Part II is work out. Show all your work. Partial credit will be given. You may use your calculator.

Part I: Multiple Choice (5 points each)

There is no partial credit. You may not use a calculator. You have 1 hour.

1. Find the area between the curves $y = x^3$ and $y = x^{1/3}$ and within the first quadrant.

- **a**. $\frac{1}{4}$
- **b**. $\frac{1}{3}$
- **c**. $\frac{1}{2}$
- **d**. $\frac{2}{3}$
- **e**. $\frac{3}{4}$

2. Find the plane tangent to the monkey saddle $z = x^3 - 3xy^2$ at the point $(2, 1, 2)$. Its z-intercept is $z =$

- **a**. 2
- **b**. 0
- **c**. -2
- **d**. -4
- **e**. -6

3. In the partial fraction expansion $\frac{2x+1}{(x^2+x+1)(x-1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+x+1}$ the coefficients are

- **a**. $A = -1, B = 0, C = 1$
- **b**. $A = 1, B = -1, C = 0$
- **c**. $A = 0, B = 1, C = 1$
- **d**. $A = 0, B = 0, C = -1$
- **e**. $A = 1, B = -1, C = -1$

4. The area in the first quadrant between the hyperbola $y = \frac{3}{x}$ and the line $y = 4 - x$ is rotated about the x-axis. Find the volume of the solid swept out.

- **a.** $\pi \int_1^3 \left(4 - x - \frac{3}{x}\right)^2 dx$
- **b.** $2\pi \int_1^3 \left(4 - x - \frac{3}{x}\right)^2 dx$
- **c.** $2\pi \int_1^3 \left(4 - x - \frac{3}{x}\right) dx$
- **d.** $\pi \int_1^3 (4x - x^2 - 3) dx$
- **e.** $\pi \int_1^3 (4 - x)^2 - \left(\frac{3}{x}\right)^2 dx$

5. Which limit does not exist?

- **a.** $\lim\limits_{(x,y)\to(0,0)} (x^2 + 2y^2)$
- **b.** $\lim\limits_{(x,y)\to(0,0)} (x^2 - 2y^2)$
- **c.** $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^4 + 2x^2y^2}{x^2 + y^2}$
- **d.** $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2 - 2y^2}{x^2 + y^2}$
- **e.** $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^4 - 2x^2y^2}{x^2 + y^2}$

6. $\int_0^1 \frac{x^3}{x^2 + 1} dx =$

- **a.** $\frac{1}{2}(1 - \ln 2)$
- **b.** $\frac{1}{2}(2 - \ln 2)$
- **c.** $\frac{1}{2}(2 + \ln 2)$
- **d.** $\frac{1}{2}(1 - \ln 2 + e)$
- **e.** $\frac{1}{2}(2 - \ln 2 + e)$

7. An airplane is circling above an airport, counterclockwise as seen from above. Thus its wings are banked with the left wing lower than the right wing. In what direction does the binormal $\vec{B}$ point?

- **a.** horizontally toward the center of the circle
- **b.** vertically up
- **c.** vertically down
- **d.** along the left wing
- **e.** along the right wing

8. Solve the initial value problem $xy' + y = e^x$ where $y(1) = e + 1$.

a. $\frac{e^x + 1}{x}$

b. $x(e^x - 1)$

c. $x(e^x + 1)$

d. $\frac{e^x}{x} + 1$

e. $xe^x - 1$

9. A conical reservoir with the point at the top is $8\,\text{ft}$ deep and 4ft in radius at the base. The liquid in the reservoir weighs $80\,\text{lb}/\text{ft}^3$. How much work is done to pump the liquid out the top of the reservoir?

a. 640π ft-lb

b. $\frac{5120}{3}\pi$ ft-lb

c. $\frac{10240}{3}\pi$ ft-lb

d. 4096π ft-lb

e. 20480π ft-lb

10. A nuclear power plant is producing a radioactive isotope X at the rate of $50\,\text{kg}/\text{yr}$. Let $N(t)\,\text{kg}$ be the amount of X at the power plant at time t. A known fact is that X decays at the rate of $75N(t)\,\text{kg}/\text{yr}$. Write the differential equation to be solved for $N(t)$.

a. $\frac{dN}{dt} = 75N - 50$

b. $\frac{dN}{dt} = 75N - 50t$

c. $\frac{dN}{dt} = 50 - 75N$

d. $\frac{dN}{dt} = 50t - 75N$

e. $\frac{dN}{dt} = -25N$

11. The function $f(x,y)$ has the values given below. Estimate $\frac{\partial f}{\partial x}(2.0, 3.1)$.

$f(2.0, 3.2) = 3$ $\quad$ $f(2.1, 3.2) = 7$ $\quad$ $f(2.2, 3.2) = 9$

$f(2.0, 3.1) = 2$ $\quad$ $f(2.1, 3.1) = 4$ $\quad$ $f(2.2, 3.1) = 8$

$f(2.0, 3.0) = 1$ $\quad$ $f(2.1, 3.0) = 2$ $\quad$ $f(2.2, 3.0) = 5$

a. 2
b. 15
c. 20
d. 25
e. 30

12. Which line is perpendicular to the plane $\left\{ \begin{array}{c} x = 2 + s - 3t \\ y = -1 - 2s + t \\ z = 3 - 2t \end{array} \right\}$

a. $\left\{ \begin{array}{c} x = 1 - 3t \\ y = 2 + t \\ z = 3 - 2t \end{array} \right\}$

b. $\left\{ \begin{array}{c} x = 2 + s \\ y = -1 - 2s \\ z = 3 \end{array} \right\}$

c. $\left\{ \begin{array}{c} x = 2t \\ y = -t \\ z = 3t \end{array} \right\}$

d. $\left\{ \begin{array}{c} x = 4 + 2t \\ y = 2 - t \\ z = -5 + 3t \end{array} \right\}$

e. $\left\{ \begin{array}{c} x = 3 + 4t \\ y = 2 + 2t \\ z = 1 - 5t \end{array} \right\}$

13. The legs of a right triangle are measured to be $6.0\,\text{in}$ and $8.0\,\text{in}$ each with a maximum error of $\pm 0.1\,\text{in}$. Hence the hypotenuse is $h = 10.0\,\text{in} \pm \Delta h$ with a maximum error of $\Delta h =$

a. $0.10\,\text{in}$
b. $0.11\,\text{in}$
c. $0.12\,\text{in}$
d. $0.13\,\text{in}$
e. $0.14\,\text{in}$

Part II: Work Out

Show all your work. Partial credit will be given.
You may use your calculator but only after 1 hour.

14. (10 points) Consider the curve $\boldsymbol{r}(t) = (1+t^2, 1-t^2, t)$. Compute each of the following:

a. velocity

$\boldsymbol{v} =$ ______________________

b. speed (Simplify.)

$|\boldsymbol{v}| =$ ______________________

c. acceleration

$\boldsymbol{a} =$ ______________________

d. curvature

$\kappa =$ ______________________

e. tangential acceleration

$a_T =$ ______________________

f. normal acceleration

$a_N =$ ______________________

15. (10 points) Compute $\int_0^{\pi/4} x\cos(2x)\,dx$

16. (10 points) An ant is crawling on the saddle surface $z = x^2 - y^2$. At time $t = 3$, the ant is at the position $(x,y,z) = (2,1,3)$ and satisfies $\frac{dx}{dt} = 0.3$ and $\frac{dy}{dt} = 0.2$. Find $\frac{dz}{dt}$ at $t = 3$.

17. (5 points) This is the direction field of a certain differential equation. Draw in the solution $y = y(x)$ which satisfies the initial condition $y(2) = -1$.

1.