Name_____

MATH 152H Final Exam Spring 2017

Sections 203/204 (circle one) P. Yasskin

1-12	/60	15	/12
13	/16	16	/12
14	/10	Total	/110

Multiple Choice: (5 points each. No part credit.)

1. Compute the arclength of the curve $y = \ln(\cos(x))$ between $x = 0$ and $x = \frac{\pi}{3}$.

a. $\ln\left(2 - \sqrt{3}\right)$

b. $\ln\left(2 + \sqrt{3}\right)$

c. $\ln\left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right)$

d. $\ln\left(\frac{1}{2} - \frac{\sqrt{3}}{2}\right)$

e. $\ln\left(\sqrt{3}\right)$

2. Find the surface area swept out when the curve $x = 1 + t^2$ $y = 1 - t^2$ is revolved around the y-axis for $0 \le t \le 2$.

a. $3\sqrt{2}\,\pi$

b. $4\sqrt{2}\,\pi$

c. $6\sqrt{2}\,\pi$

d. $12\sqrt{2}\,\pi$

e. $24\sqrt{2}\,\pi$

3. Find the area between $y = x^2 - 4x$ and $y = 2x - x^2$.

a. 1
b. 3
c. 9
d. 12
e. 18

4. Compute $\int_4^5 \frac{1}{x^2 - 5x + 6}\, dx$

a. $2\ln 2 - \ln 3$
b. $\ln 3 - 2\ln 2$
c. $\ln 3$
d. $2\ln 3 - \ln 2$
e. $\ln 2 - 2\ln 3$

5. The integral $\int_2^\infty \frac{1}{x^2 + x}\, dx$

a. diverges by comparison with $\int_2^\infty \frac{1}{x}\, dx$
b. converges by comparison with $\int_2^\infty \frac{1}{x}\, dx$
c. diverges by comparison with $\int_2^\infty \frac{1}{x^2}\, dx$
d. converges by comparison with $\int_2^\infty \frac{1}{x^2}\, dx$
e. None of the above

6. Compute $\int_4^5 \frac{1}{x^2\sqrt{x^2-16}}\,dx$

a. $\frac{3}{80}$

b. $\frac{9}{80}$

c. $\frac{3}{160}$

d. $\frac{9}{40}$

e. $\frac{3}{40}$

7. Compute $\int_0^1 3x^2 \arctan x\,dx$

a. $\frac{\pi}{2} + \ln 2 - 1$

b. $\frac{\pi}{2} + \frac{1}{2}\ln 2 + \frac{1}{2}$

c. $\frac{\pi}{4} + \ln 2 - 1$

d. $\frac{\pi}{4} + \frac{1}{2}\ln 2 - \frac{1}{2}$

e. $\frac{\pi}{4} + \frac{1}{2}\ln 2 + \frac{1}{2}$

8. Solve the initial value problem $\frac{dy}{dx} = 1 + 2x + y^2 + 2xy^2$ with $y(1) = 0.$ What is $y(2)$?
HINT: Factor.

a. $\tan(1)$

b. $\tan(2)$

c. $\tan(4)$

d. $\frac{\pi}{4}$

e. $\frac{\pi}{2}$

9. Compute $\sum_{n=1}^{\infty} \frac{3^{2n}}{2^{3n}}$.

a. -9

b. -8

c. 8

d. 9

e. divergent

10. Compute $\sum_{n=1}^{\infty} \left[\cos\left(\frac{\pi}{n}\right) - \cos\left(\frac{\pi}{n+1}\right) \right]$.

a. -2

b. -1

c. 1

d. 2

e. divergent

11. Compute $\lim_{x\to 0} \frac{e^{x^2} - \cos x}{x^2}$

a. $\frac{1}{2}$

b. $\frac{3}{2}$

c. $\frac{5}{2}$

d. $\frac{11}{24}$

e. $\frac{13}{24}$

12. The series $\sum_{n=2}^{\infty} \frac{1}{n^2 - n}$

a. diverges by a Simple Comparison with $\sum_{n=2}^{\infty} \frac{1}{n}$

b. converges by a Simple Comparison with $\sum_{n=2}^{\infty} \frac{1}{n}$

c. converges by a Simple Comparison with $\sum_{n=2}^{\infty} \frac{1}{n^2}$

d. diverges by a Limit (but not Simple) Comparison with $\sum_{n=2}^{\infty} \frac{1}{n^2}$

e. converges by a Limit (but not Simple) Comparison with $\sum_{n=2}^{\infty} \frac{1}{n^2}$

Work Out: (Points indicated. Part credit possible. Show all work.)

13. $(16$ points$)$ Let $X(t)$ be the amount of a radio active element X present at a reactor.

The element X is produced at $100\frac{\text{kg}}{\text{yr}}$ and decays with a half-life of 20 yrs.

If we start with no element X on hand, then $X(t)$ satisfies the initial value problem

$$\frac{dX}{dt} = 100 - \frac{\ln 2}{20}X \qquad \text{with} \quad X(0) = 0$$

a. Solve the initial value problem:

b. How much of element X is present after 20 yrs?

14. (10 points) Use a Maclaurin polynomial to estimate $\sin(1)$ to within 10^{-5}.
What theorem guarantees the error in the approximation is less than 10^{-5}?
Do not add up the terms. No decimals!

Note: $0! = 1$ $1! = 1$ $3! = 6$ $5! = 120$ $7! = 5040$ $9! = 362\,880$

15. (12 points) Use the Ratio Test to find the radius of convergence of each of the following series:

a. $\sum_{n=1}^{\infty} \frac{3^n}{n}(x-4)^n$

$R =$

b. $\sum_{n=1}^{\infty} \frac{n}{3^n}(x-4)^n$

$R =$

c. $\sum_{n=1}^{\infty} \frac{3^n}{n!}(x-4)^n$

$R =$

d. $\sum_{n=1}^{\infty} \frac{n!}{3^n}(x-4)^n$

$R =$

16. (12 points) The series $\sum_{n=1}^{\infty} \frac{1}{2^n \sqrt{n}}(x-3)^n$ has radius of convergence $R = 2$.

Find its interval of convergence.

Left Endpoint: $x =$ Series at Left Endpoint: $\sum$

Name and Apply Test for Convergence:

Conclusion (Circle one)

Convergent

Divergent

Right Endpoint: $x =$ Series at Right Endpoint: $\sum$

Name and Apply Test for Convergence:

Conclusion (Circle one)

Convergent

Divergent

Interval of Convergence: