Name________________________________

MATH 172 Exam 3 Spring 2018

Sections 501/502 (circle one) P. Yasskin

Multiple Choice: (5 points each. No part credit.)

1. Which of the following polar coordinates is NOT the point $\left(-3, \frac{\pi}{3}\right)$?

a. $(r,\theta) = \left(-3, \frac{-5\pi}{3}\right)$

b. $(r,\theta) = \left(3, \frac{2\pi}{3}\right)$

c. $(r,\theta) = \left(-3, \frac{7\pi}{3}\right)$

d. $(r,\theta) = \left(3, \frac{-2\pi}{3}\right)$

e. $(r,\theta) = \left(3, \frac{4\pi}{3}\right)$

2. Find the arc length of the piece of the cardioid $r = 2 - 2\cos\theta$ in the upper half-plane.

a. 1

b. 2

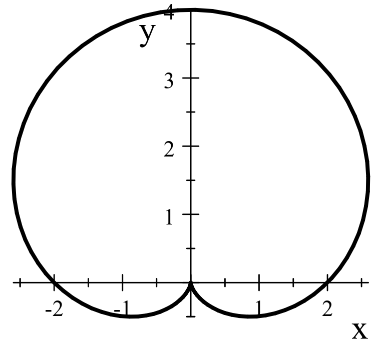
c. 4

d. 8

e. 16

1-11	/55	13	/20
12	/10	14	/20
		Total	/105

3. Find the area inside the cardioid $r = 2 + 2\sin\theta$.



a. $A = \pi$

b. $A = 2\pi$

c. $A = 3\pi$

d. $A = 4\pi$

e. $A = 6\pi$

4. $\lim\limits_{n\to\infty} \dfrac{n^2 + n}{2n^2 + (-1)^n} =$

a. 1

b. 2

c. $\frac{1}{2}$

d. $\frac{1}{3}$

e. $\frac{2}{3}$

5. $\lim\limits_{n\to\infty} \left(\sqrt{n^2 - 4n} - n\right)$

a. -2

b. 0

c. 2

d. 4

e. ∞

6. $\lim_{n\to\infty} \sqrt{n}^{\,(3/\ln n)} =$

a. 0
b. e
c. e^3
d. 1
e. 3

7. $\sum_{n=1}^{\infty} \frac{3^{2n}}{2^{4n+1}} =$

a. $\frac{3}{7}$
b. $\frac{3}{14}$
c. $\frac{9}{7}$
d. $\frac{9}{14}$
e. diverges

8. $\sum_{n=1}^{\infty} \left(\frac{n-1}{n} - \frac{n}{n+1} \right) =$

a. -1
b. 0
c. 1
d. 2
e. diverges

9. $\sum_{n=1}^{\infty} \frac{n}{n^2+1} =$

a. -1
b. 0
c. 1
d. 2
e. diverges

10. $\sum_{n=1}^{\infty} \frac{n-1}{n}$

a. -1
b. 0
c. 1
d. 2
e. diverges

11. Given a sequence a_n, suppose the partial sum is $S_k = \sum_{n=2}^{k} a_n = \ln\left(\frac{k}{k+1}\right)$. What is a_n?

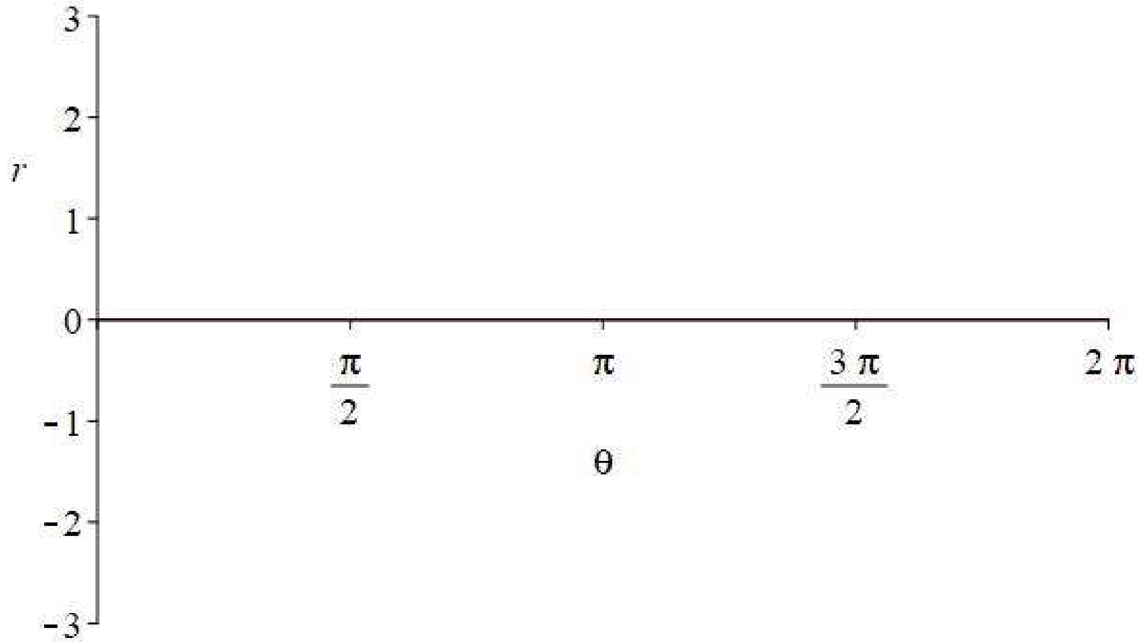
HINT: What is S_{k-1}? What is the relation between S_k and S_{k-1}?

a. $a_n = \ln\left(\frac{n^2}{n^2-1}\right)$
b. $a_n = \ln\left(\frac{n^2}{n^2+1}\right)$
c. $a_n = \ln\left(\frac{n^2-n}{n^2+n}\right)$
d. $a_n = \ln\left(\frac{n^2+n}{n^2-n}\right)$
e. Insufficient information to determine a_n.

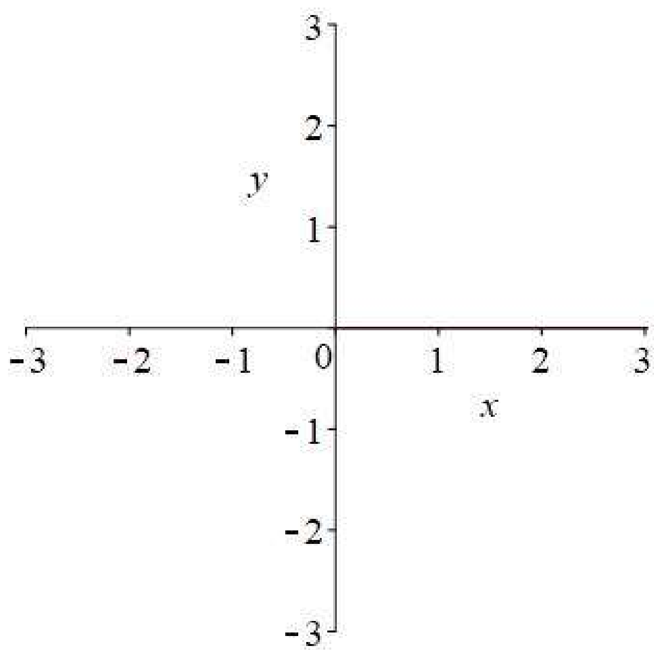
Work Out: (Points indicated. Part credit possible. Show all work.)

12. (10 points) Consider the polar curve $r = 1 + \sqrt{2}\cos\theta$. Approximate $\sqrt{2} \approx 1.4$.

a. Plot a rectangular graph of r as a function of θ with r vertical and θ horizontal.
(You don't need to be too precise, but be careful with the values at $\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$.)



b. Plot a polar graph of r as a function of θ with r radial and θ angular.
(You don't need to be too precise, but be careful with the values at $\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$.)



13. (20 points) Use a Comparison Theorem to determine whether each of the following series converges or diverges. Clearly state the comparison series, why the comparison series converges or diverges and why the original series converges or diverges. For each conclusion, name the Convergence Test.

a. $\sum_{n=1}^{\infty} \frac{1}{n + n^2}$

b. $\sum_{n=1}^{\infty} \frac{1}{n + \sqrt{n}}$

14. (20 points) Consider the recursively defined sequence

$$a_1 = 3 \qquad a_{n+1} = \sqrt{6a_n - 8}$$

a. Write out the first 3 terms of the sequence:

$a_1 =$ ________ $a_2 =$ ________________________ $a_3 =$ ____________________

b. Use mathematical induction to prove the sequence is increasing.
(Be sure to state the formula you need to prove.)

c. Use mathematical induction to prove the sequence is bounded above by 6.
(Be sure to state the formula you need to prove.)

(continued)

d. State a theorem which guarantees the sequence converges.

e. Find the limit of the sequence.