

Name \_\_\_\_\_

MATH 251      Exam 1 Version H      Fall 2017  
Sections 200      P. Yasskin

1-9	/54	11	/16
10	/33	Total	/103

Multiple Choice: (6 points each. No part credit.)

1. The points  $A = (2, -3, 4)$  and  $B = (4, 1, 0)$  are the endpoints of the diameter of a sphere. What is the radius of the sphere?

- a. 6
- b. 5
- c. 4
- d. 3
- e. 2

2. Consider the permutation  $p = (2, 6, 4, 1, 3, 5)$ . Find its inverse and parity (odd vs. even).

- a.  $\bar{p} = (4, 1, 5, 3, 6, 2)$        $\varepsilon_p = 1$
- b.  $\bar{p} = (4, 1, 5, 3, 6, 2)$        $\varepsilon_p = -1$
- c.  $\bar{p} = (2, 6, 3, 5, 1, 4)$        $\varepsilon_p = 1$
- d.  $\bar{p} = (2, 6, 3, 5, 1, 4)$        $\varepsilon_p = -1$
- e.  $\bar{p} = (5, 3, 1, 4, 6, 2)$        $\varepsilon_p = 1$

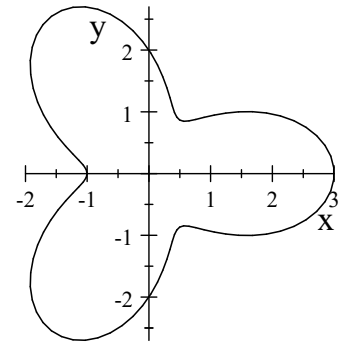
3. Find the angle between the normals to the planes  $3x + 2y - 4z = 3$  and  $2x - y + z = 2$ .

- a.  $0^\circ$
- b.  $30^\circ$
- c.  $45^\circ$
- d.  $60^\circ$
- e.  $90^\circ$

4. Duke Skywater pushes an asteroid from the point  $P = (2, -3, 5)$  to the point  $Q = (5, -1, 4)$  by the force  $\vec{F} = (4, 1, 2)$ . Find the work done to move the asteroid.
- 16
  - 12
  - 6
  - 4
  - 2

5. The plot at the right is which polar equation?

- $r = 2 + \cos 3\theta$
- $r = 2 - \cos 3\theta$
- $r = 1 + 2 \cos 3\theta$
- $r = 1 - 2 \cos 3\theta$
- $r = 1 + \cos 3\theta$



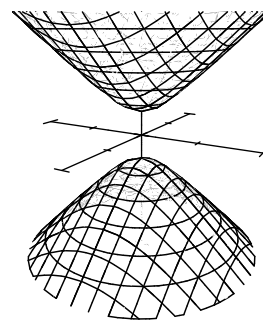
6. In  $\mathbb{R}^4$ , find a vector perpendicular to the hyperplane containing the 4 points  $P = (2, 1, 4, 1)$ ,  $Q = (-1, 3, 2, 1)$ ,  $R = (3, 1, 2, 2)$  and  $S = (3, 1, 4, 1)$
- $(-1, 2, 2, -4)$
  - $(1, 2, 2, 4)$
  - $(0, 2, 2, 4)$
  - $(1, -2, 2, -4)$
  - $(0, -2, 2, -4)$

7. If  $|\vec{u}| = 2$  and  $|\vec{v}| = 5$  and  $\vec{u} \cdot \vec{v} = 6$  find  $|\vec{u} \times \vec{v}|$ .

- a. 8
- b. 6
- c. 4
- d. 2
- e. 0

8. The plot at the right is the graph of which equation?

- a.  $x^2 + y^2 - z^2 = 1$
- b.  $x^2 + y^2 - z^2 = 0$
- c.  $x^2 + y^2 - z^2 = -1$
- d.  $x^2 + y^2 - z = 1$
- e.  $x^2 + y^2 - z = -1$



9. Find the point where the line  $(x, y, z) = \vec{r}(t) = (2t + 1, t - 1, 2t - 1)$  intersects the plane  $3x + 2y + z = 20$ .

At this point  $x + y + z =$

- a. -6
- b. -1
- c. 4
- d. 9
- e. 13

Work Out: (Points indicated. Part credit possible. Show all work.)

10. (33 points) For the parametric curve  $\vec{r}(t) = \left(\frac{2}{t}, 6t, 3t^3\right)$  compute each of the following:

a. (3 pts) velocity  $\vec{v}$

$$\vec{v} = \underline{\hspace{10cm}}$$

b. (3 pts) acceleration  $\vec{a}$

$$\vec{a} = \underline{\hspace{10cm}}$$

c. (3 pts) jerk  $\vec{j}$

$$\vec{j} = \underline{\hspace{10cm}}$$

d. (3 pts) speed  $|\vec{v}|$  (Simplify!)

HINT: The quantity inside the square root is a perfect square.

$$|\vec{v}| = \underline{\hspace{10cm}}$$

e. (3 pts) tangential acceleration  $a_T$

$$a_T = \underline{\hspace{10cm}}$$

f. (4 pts) unit binormal  $\hat{B}$  (Do this last.)

$$\hat{B} = \underline{\hspace{10cm}}$$

Recall:  $\vec{r}(t) = \left(\frac{2}{t}, 6t, 3t^3\right)$

g. (2 pts) the values of  $t$  where the curve passes thru the points

$A = (2, 6, 3)$

$t = \underline{\hspace{2cm}}$

$B = (1, 12, 24)$

$t = \underline{\hspace{2cm}}$

h. (4 pts) arc length between  $(2, 6, 3)$  and  $(1, 12, 24)$ ,  $L = \int_{(2,6,3)}^{(1,12,24)} ds$

$L = \underline{\hspace{2cm}}$

i. (4 pts) A wire has the shape of this curve between  $(2, 6, 3)$  and  $(1, 12, 24)$ . Find the mass of the wire if the linear mass density is  $\rho = \frac{1}{6}xz$ .

(Don't simplify the answer.)

$M = \underline{\hspace{2cm}}$

j. (4 pts) A wire has the shape of this curve. Find the work done by the force  $\vec{F} = (z, y, x)$  which pushes a bead along the wire from  $(2, 6, 3)$  to  $(1, 12, 24)$ .

$W = \underline{\hspace{2cm}}$

11. (16 points) Are the following lines parallel, intersecting or skew? If they intersect, find the point of intersection.

a. Line 1:  $\vec{r}_1(t) = (t + 2, t - 2, 2t - 1)$

Line 2:  $\vec{r}_2(t) = (t + 1, 2t - 6, 2t - 1)$

b. Line 1:  $\vec{r}_1(t) = (t + 2, t - 2, 2t + 1)$

Line 2:  $\vec{r}_2(t) = (t + 1, 2t - 6, 2t - 1)$