Name________________________

1-13	/65	15	/15
14	/15	16	/15
		Total	/110

MATH 251 Exam 2 Version B Fall 2017

Sections 515 P. Yasskin

Multiple Choice: (5 points each. No part credit.)

1. Find the equation of the plane tangent to $z = x^2 y + xy^2$ at $(x,y) = (1,2)$.
The z-intercept is:

a. $c = -24$

b. $c = -12$

c. $c = 12$

d. $c = -6$

e. $c = 6$

2. Find the plane tangent to the ellipsoid $36x^2 + 9y^2 + 4z^2 = 108$ at the point $(x,y,z) = (1,2,3)$.

a. $6x + 12y + 18z = 84$

b. $\dfrac{x}{6} + \dfrac{y}{12} + \dfrac{z}{18} = \dfrac{1}{2}$

c. $6x + 3y + 2z = 18$

d. $\dfrac{x}{6} + \dfrac{y}{3} + \dfrac{z}{2} = \dfrac{7}{3}$

e. $36x + 9y + 4z = 18$

3. If $f(x,y) = x\cos(y) + y\sin(x)$, which of the following is INCORRECT?

a. $f_x = \cos(y) + y\cos(x)$

b. $f_y = -x\sin(y) + \sin(x)$

c. $f_{xx} = -y\sin(x)$

d. $f_{xy} = -\sin(y) + \cos(x)$

e. $f_{yx} = \sin(y) + \cos(x)$

4. A support beam is constructed using four struts whose lengths are w, x, y and z. The strength of the beam is $S = w^2x + y^2z$. If the current lengths are $w = 1$, $x = 3$, $y = 2$ and $z = 1$, then the current strength is $S = 1^2 3 + 2^2 1 = 7$. Use differentials (i.e. the linear approximation) to estimate how much the strength increases, ΔS, if the lengths increase by $\Delta w = 0.1$, $\Delta x = 0.2$, $\Delta y = 0.2$ and $\Delta z = 0.3$.

a. 0.8

b. 1.4

c. 2.1

d. 2.8

e. 3.5

5. In the coutour plot at the right, which point is the saddle point?

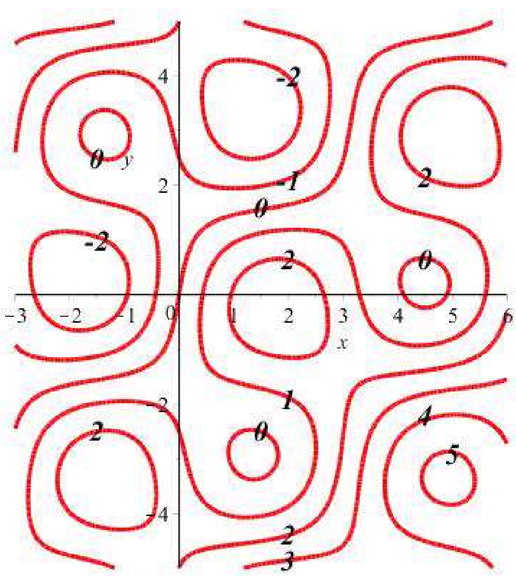
a. $(3.5, 1.5)$

b. $(5, -1)$

c. $(5, -3.5)$

d. $(1.5, 3.5)$

e. $(-1.5, -3.5)$



6. Use the linear approximation to the function $f(x,y) = \sqrt{x^2 + y^2}$ to estimate $\sqrt{3.9^2 + 3.2^2}$.

a. 5.02

b. 5.04

c. 5.10

d. 5.40

e. 5.73

7. A weather balloon is currently located at $(x,y,z) = (20,30,10)$ and has velocity $\vec{v} = (3,1,2)$. At the current time, it measures that the pressure is $P = .96$ atm and has gradient

$$\vec{\nabla}P = \left\langle \frac{\partial P}{\partial x}, \frac{\partial P}{\partial y}, \frac{\partial P}{\partial z} \right\rangle = \langle .01, .02, .03 \rangle$$

Find the rate of change of the pressure as seen aboard the balloon.

a. 0.08

b. 0.09

c. 0.10

d. 0.11

e. 0.12

8. Ham Duet is flying the Centurion Eagle through a nebula where the density of cloaking sparkles is $\delta = xyz$. If Ham's current position is $P = (1,1,2)$, find the rate of change of the density in the direction toward the point $Q = (-1,3,3)$.

a. $\frac{5}{3}$

b. $\frac{4}{3}$

c. 1

d. $\frac{2}{3}$

e. $\frac{1}{3}$

9. Ham Duet is flying the Centurion Eagle through a nebula where the density of cloaking sparkles is $\delta = xyz$. If Ham's current position is $P = (1,1,2)$, in what unit vector direction should he travel to increase the cloaking sparkles as fast as possible?

a. $\left\langle \frac{2}{3}, \frac{2}{3}, \frac{1}{3} \right\rangle$

b. $\left\langle -\frac{2}{3}, \frac{2}{3}, -\frac{1}{3} \right\rangle$

c. $\left\langle -\frac{2}{3}, \frac{2}{3}, \frac{1}{3} \right\rangle$

d. $\left\langle \frac{2}{3}, -\frac{2}{3}, -\frac{1}{3} \right\rangle$

e. $\left\langle -\frac{2}{3}, -\frac{2}{3}, \frac{1}{3} \right\rangle$

10. If $\vec{F} = \langle x^2y, y^2z, z^2x \rangle$, then $\vec{\nabla} \cdot \vec{F} =$

a. $-y^2 - z^2 - x^2$

b. $2xy - 2yz + 2zx$

c. $2xy + 2yz + 2zx$

d. $\langle 2xy, -2yz, 2zx \rangle$

e. $\langle 2xy, 2yz, 2zx \rangle$

11. If $\vec{F} = \langle x^2y, y^2z, z^2x \rangle$, then $\vec{\nabla} \times \vec{F} =$

a. $-y^2 + z^2 - x^2$

b. $\langle 2xy, -2yz, 2zx \rangle$

c. $\langle 2xy, 2yz, 2zx \rangle$

d. $\langle -y^2, -z^2, -x^2 \rangle$

e. $\langle -y^2, z^2, -x^2 \rangle$

12. If $\vec{F} = \langle x^2y, y^2z, z^2x \rangle$, then $\vec{\nabla} \cdot \vec{\nabla} \times \vec{F} =$

a. $-y^2 + z^2 - x^2$

b. $-y^2 - z^2 - x^2$

c. $2y + 2z + 2x$

d. $2y - 2z + 2x$

e. 0

13. Find a scalar potential, f, for the vector field $\vec{F} = \langle yz + 6x, xz - 4y, xy \rangle$.
Then $f(2,2,2) - f(1,1,1) =$

a. 15

b. 10

c. 5

d. 2

e. 1

Work Out: (15 points each. Part credit possible. Show all work.)

14. (15 points) The Ideal Gas Law says the Pressure, P, Volume, V, and Temperature, T, are related by $PV = kT$. Currently, a particular sample of ideal gas has the parameters:

$P = 0.9$ atm $\qquad V = 600\ \text{cm}^3 \qquad$ and $\qquad T = 270°$K

a. First find the constant k.

b. If the volume is increasing at $\dfrac{dV}{dt} = \dfrac{8\ \text{cm}^3}{\text{hr}}$ while the temperature is increasing at $\dfrac{dT}{dt} = \dfrac{3°\text{K}}{\text{hr}}$, at what rate, $\dfrac{dP}{dt}$, is the pressure changing?
Is the pressure increasing or decreasing?

15. (15 points) Find all critical points of the function $f(x,y) = x^3 - 12x + 3xy^2$.
Then use the second derivative test to classify each as a
local minimum, local maximum or saddle or say the test fails.

16. (15 points) Find the point on the plane $2x - 2y - z = 18$ that is closest to the origin.
You may use either the Eliminate a Variable method or the Lagrange Multiplier method.