Name______________________________

1-10	/50	13	/15
11	/ 5	14	/15
12	/20	Total	/105

MATH 251 Final Exam Version A Fall 2017

Sections 515 P. Yasskin

Multiple Choice: (5 points each. No part credit.)

1. A wire has the shape of the helix curve $\vec{r}(\theta) = (3\cos\theta, 3\sin\theta, 4\theta)$ for $0 \le \theta \le \pi$ and has linear density $\delta = 2y$. Find the total mass of the wire.

a. 80

b. 60

c. 40

d. 20

e. 10

2. A wire has the shape of the helix curve $\vec{r}(\theta) = (3\cos\theta, 3\sin\theta, 4\theta)$ for $0 \le \theta \le \pi$ and has linear density $\delta = 2y$. Find the y-component of the center of mass of the wire.

a. $\dfrac{3\pi}{4}$

b. $\dfrac{4}{3\pi}$

c. 45π

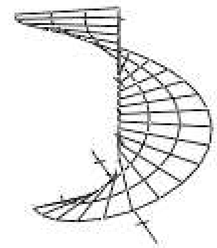
d. $\dfrac{1}{45\pi}$

e. 3π

3. The spiral ramp shown at the right may be parametrized by

$$\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, \theta)$$

for $0 \le r \le 3$ and $0 \le \theta \le 2\pi$.



Find the total mass, if the surface density is

$$\delta = \sqrt{x^2 + y^2}$$

a. $\frac{2\pi}{3}(5^{3/2} - 1)$

b. $\frac{2\pi}{3}5^{3/2}$

c. $\frac{2\pi}{3}(10^{3/2} - 1)$

d. $\frac{2\pi}{3}10^{3/2}$

e. $\frac{2\pi}{3}(10^{3/2} - 5^{3/2})$

4. Consider the spiral ramp described in the previous problem.
Find the flux of the vector field $\vec{F} = (0,0,z)$ **upward** through the spiral ramp.

a. $-9\pi^2$

b. $-4\pi^2$

c. 0

d. $4\pi^2$

e. $9\pi^2$

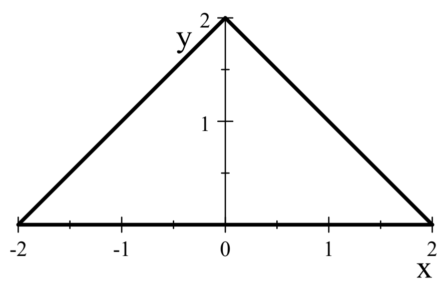
5. Compute $\int_A^B \vec{F} \cdot d\vec{s}$ for $\vec{F} = (2x + y, x + 2y)$ along the line segment from $A = (2,2)$ to $B = (3,3)$.
Hint: Find a scalar potential.

a. 15
b. 6
c. 0
d. -6
e. -15

6. Compute $\int_A^B \vec{F} \cdot d\vec{s}$ for $\vec{F} = (-y, x, 2)$ along the helix $\vec{r}(\theta) = (4\cos\theta, 4\sin\theta, 3\theta)$ from $A = (4,0,0)$ to $B = (4,0,6\pi)$.

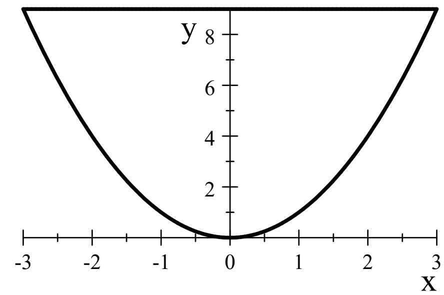
a. 0
b. 40π
c. 42π
d. 44π
e. 46π

7. Compute $\oint_{\partial T} (\sin x + 5y)\,dx + (3x + \cos y)\,dy$
clockwise around the complete boundary of the triangle shown at the right.
Hint: Use a Theorem.



a. 12
b. 8
c. 0
d. -8
e. -12

8. Compute $\oint \vec{F} \cdot d\vec{s}$ for $\vec{F} = (x^2 y, 2x^3)$ along the piece of the parabola $y = x^2$ from $(-3,9)$ to $(3,9)$ followed by the line segment from $(3,9)$ back to $(-3,9)$.
Hint: Use Green's Theorem.



a. 0
b. 81
c. 162
d. 324
e. 405

9. Compute $\iint_{\partial C} \vec{F} \cdot d\vec{S}$ over the complete surface of the cylinder $x^2 + y^2 \le 4$ for $0 \le z \le 3$ oriented out from the cylinder for $\vec{F} = (xz, yz, z^2)$.
Hint: Use Gauss' Theorem.

a. 24π
b. 36π
c. 72π
d. 144π
e. 288π

10. Sketch the region of integration for the integral $\int_0^2 \int_{x^2}^4 x\cos(y^2)\, dy\, dx$ in problem (11).
Select its value here:

a. $\frac{1}{2}\sin 16$

b. $\frac{1}{4}\sin 16$

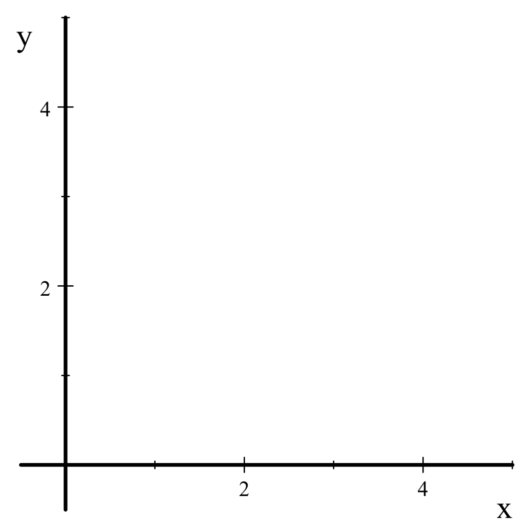
c. $\frac{1}{2}\sin 4$

d. $\frac{1}{4}\sin 4$

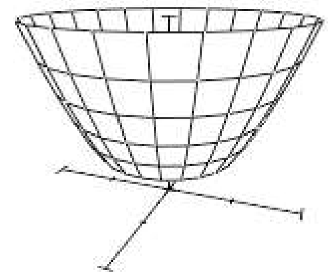
e. $\frac{1}{4}\sin 2$

Work Out: (Points indicated. Part credit possible. Show all work.)

11. (5 points) Sketch the region of integration for the integral $\int_0^2 \int_{x^2}^4 x\cos(y^2)$.
Shade in the region.
Compute its value in problem (10).



12. (20 points) Verify Stokes' Theorem $\iint\limits_P \vec{\nabla} \times \vec{F} \cdot d\vec{S} = \oint\limits_{\partial P} \vec{F} \cdot d\vec{s}$



for the vector field $\vec{F} = (-2yz, 2xz, z^2)$ and the **surface** which is the piece of the paraboloid P given by $z = x^2 + y^2$ between $z = 1$ and $z = 4$ oriented up and in.
Notice that the boundary of P is two circles.

Be sure to check orientations. Use the following steps:

a. The paraboloid may be parametrized by $\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, r^2)$ for $1 \le r \le 2$.

$\vec{e}_r =$

$\vec{e}_\theta =$

$\vec{N} =$ (Check the orientation)

$\vec{\nabla} \times \vec{F} =$

$\vec{\nabla} \times \vec{F}\Big|_{\vec{R}(r,\theta)} =$

$\vec{\nabla} \times \vec{F} \cdot \vec{N} =$

$\iint\limits_P \vec{\nabla} \times \vec{F} \cdot d\vec{S} =$

Recall $\vec{F} = (-2yz, 2xz, z^2)$

b. Parametrize the upper circle U and compute the line integral.

$\vec{r}(\theta) =$

$\vec{v}(\theta) =$ (Check the orientation)

$\vec{F}\big|_{\vec{r}(\theta)} =$

$\oint_U \vec{F} \cdot d\vec{s} =$

c. Parametrize the lower circle L and compute the line integral.

$\vec{r}(\theta) =$

$\vec{v}(\theta) =$ (Check the orientation)

$\vec{F}\big|_{\vec{r}(\theta)} =$

$\oint_L \vec{F} \cdot d\vec{s} =$

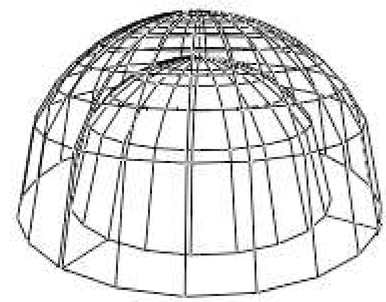
d. Combine $\oint_U \vec{F} \cdot d\vec{s}$ and $\oint_L \vec{F} \cdot d\vec{s}$ to get $\oint_{\partial C} \vec{F} \cdot d\vec{s}$.

13. (15 points) (Also replaces Exam 3 #12.)

Find the mass of the solid between the hemispheres

$z = \sqrt{9 - x^2 - y^2}$ and $z = \sqrt{16 - x^2 - y^2}$

for $z \geq 0$ if the density is $\delta = \dfrac{1}{x^2 + y^2 + z^2}$.



14. (15 points) (Also replaces Exam 3 #13.)

Find the **centroid** of the **solid** inside the paraboloid $z = x^2 + y^2$ for $1 \leq z \leq 4$.

Hint: Put the differentials in the order $dr\,dz\,d\theta$.