Name________________________ ID____________ Section______

1-7	/49
8	/25
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MATH 253 EXAM 3 Fall 1998

Sections 501-503 P. Yasskin

Multiple Choice: (7 points each)

1. If $F = \left(yz\cos x,\ y\sin x,\ z\sin x\right)$ then $\vec{\nabla}\bullet\vec{F} =$

a. $-yz\sin x$

b. $(2-yz)\sin x$

c. $\left(-yz\sin x,\ -\sin x,\ \sin x\right)$

d. $\left(0,\ (z-y)\cos x,\ (y-z)\cos x\right)$

e. $\left(-yz\sin x,\ \sin x,\ \sin x\right)$

2. If $F = \left(yz\cos x,\ y\sin x,\ z\sin x\right)$ then $\vec{\nabla}\times\vec{F} =$

a. $\left(0,\ (y-z)\cos x,\ (y-z)\cos x\right)$

b. $2(y-z)\sin x$

c. $\left(-yz\sin x,\ -\sin x,\ \sin x\right)$

d. $\left(0,\ (z-y)\cos x,\ (y-z)\cos x\right)$

e. $\vec{0}$

3. Find a scalar potential for $F = \left(yz\cos x,\ y\sin x,\ z\sin x\right)$.

a. $-yz\sin x$

b. $yz\sin x$

c. $\left(yz\sin x,\ \dfrac{y^2}{2}\sin x,\ \dfrac{z^2}{2}\sin x\right)$

d. $yz\sin x + \dfrac{y^2}{2}\sin x + \dfrac{z^2}{2}\sin x$

e. Does Not Exist

4. Compute the line integral $\int_{A \atop H}^{B} \vec{F} \bullet d\vec{s}$ of the vector field $\vec{F} = (y, -x, z)$ along the helix H parametrized by $\vec{r}(t) = (3\cos\theta, 3\sin\theta, \theta)$ between $A = (3,0,0)$ and $B = (-3,0,3\pi)$.

a. $2\pi^2 - 18\pi$

b. 2π

c. 3π

d. $\frac{9\pi^2}{2} - 27\pi$

e. $\frac{9\pi^2}{2}$

5. Find the total mass of the helix H parametrized by $\vec{r}(t) = (3\cos\theta, 3\sin\theta, \theta)$ between $A = (3,0,0)$ and $B = (-3,0,3\pi)$ if the linear mass density is $\rho = 3 + 2z$.

a. $9(\pi + \pi^2)$

b. $27(\pi + \pi^2)$

c. $9\sqrt{10}(\pi + \pi^2)$

d. $\sqrt{10}(6\pi + 4\pi^2)$

e. $6\pi + 4\pi^2$

6. Compute $\oint (4x - 3y)\,dx + (3x - 2y)\,dy$ counterclockwise around the edge of the triangle with vertices $(0,0)$, $(0,3)$ and $(2,0)$.
(HINT: Use Green's Theorem.)

a. 3
b. 6
c. 9
d. 12
e. 18

7. Compute $\iint\limits_P \vec{\nabla} \times \vec{F} \bullet d\vec{S}$ for the vector field $\vec{F} = (-y, x, z)$ over the paraboloid $z = x^2 + y^2$ for $z \le 9$ with normal pointing in and up.
(HINT: Use Stokes' Theorem.)

a. 2π
b. 3π
c. 4π
d. 9π
e. 18π

8. (25 points) Green's Theorem states that if R is a nice region in the plane and ∂R is its boundary curve traversed counterclockwise then

$$\iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx\,dy = \oint_{\partial R} P\,dx + Q\,dy$$

Verify Green's Theorem if $P = -x^2 y$ and $Q = xy^2$ and R is the region inside the circle $x^2 + y^2 = 4$.

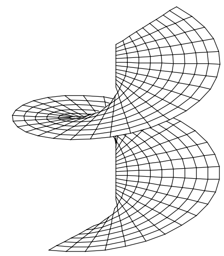
a. (5 pts) Compute $\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}$. (HINT: Use rectangular coordinates.)

b. (10 pts) Compute $\iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx\,dy$.

(HINT: Switch to polar coordinates and don't forget the Jacobian.)

c. (10 pts) Compute $\oint_{\partial R} P\,dx + Q\,dy$. (HINT: Parametrize the boundary circle.)

9. (15 points) The spiral ramp at the right may be parametrized as



$$\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, \theta)$$

for $0 \le r \le 2$ and $0 \le \theta \le 3\pi$.

Compute $\iint \sqrt{x^2 + y^2}\ dS$ over this spiral ramp.

10. (25 points) Gauss' Theorem states that if V is a solid region and ∂V is its boundary surface with outward normal then

$$\iiint_V \vec{\nabla} \bullet \vec{F}\, dV = \iint_{\partial V} \vec{F} \bullet d\vec{S}$$

Verify Gauss' Theorem if $F = (xz^2, yz^2, z^3)$ and V is the solid region above the paraboloid $z = x^2 + y^2$ below the plane $z = 4$. Notice that ∂V consists of two parts: a piece of the paraboloid P and a disk D.

a. (7 pts) Compute $\iiint_V \vec{\nabla} \bullet \vec{F}\, dV$. (HINTS: Compute $\vec{\nabla} \bullet \vec{F}$ in rectangular coordinates. Integrate in cylindrical coordinates.)

$\vec{\nabla} \bullet \vec{F} =$

$\iiint_V \vec{\nabla} \bullet \vec{F}\, dV =$

b. (8 pts) Compute $\iint_P \vec{F} \bullet d\vec{S}$. (HINT: Here is the parametrization of the paraboloid.)

$\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, r^2)$

$\vec{R}_r =$

$\vec{R}_\theta =$

$\vec{N} =$

$\vec{F}\left(\vec{R}(r,\theta)\right) =$

$\iint_P \vec{F} \bullet d\vec{S} =$

c. (7 pts) Compute $\iint_D \vec{F} \bullet d\vec{S}$. (HINT: You parametrize the disk.)

$\vec{R}(r,\theta) =$

$\vec{R}_r =$

$\vec{R}_\theta =$

$\vec{N} =$

$\vec{F}\left(\vec{R}(r,\theta)\right) =$

$\iint_D \vec{F} \bullet d\vec{S} =$

d. (3 pts) Combine $\iint_P \vec{F} \bullet d\vec{S}$ and $\iint_D \vec{F} \bullet d\vec{S}$. to obtain $\iint_{\partial V} \vec{F} \bullet d\vec{S}$.
Be sure to discuss the orientations of the surfaces and to give a formula before you plug in numbers.

$\iint_{\partial V} \vec{F} \bullet d\vec{S} =$