Name__________________________

MATH 253 Exam 2 Fall 2016

Sections 201/202 P. Yasskin

| | |
|---|---|
| 1-8 | /40 |
| 9 | /30 |
| 10 | /16 |
| 11 | /15 |
| E.C. | / 5 |
| Total | /106 |

Multiple Choice: (5 points each. No part credit.)

**1**. The function $f = \sin x \cos y$ has a critical point at $(x,y) = \left(\pi, \frac{\pi}{2}\right)$.
Use the Second Derivative Test to classify this critical point.

**a**. Local Minimum
**b**. Local Maximum
**c**. Inflection Point
**d**. Saddle Point
**e**. Test Fails

**2**. Find the volume of the solid under $z = 2x^2 y$ above the region in the $xy$-plane between $y = x$ and $y = x^2$.

**a**. $\frac{1}{12}$
**b**. $\frac{35}{12}$
**c**. $\frac{12}{35}$
**d**. $\frac{1}{35}$
**e**. $\frac{2}{35}$

3. Compute $\iint \sin(x^2)\,dx\,dy$ over the triangle with vertices $(0,0)$, $(\sqrt{\pi},0)$, $(\sqrt{\pi},\sqrt{\pi})$.

a. $-\pi$

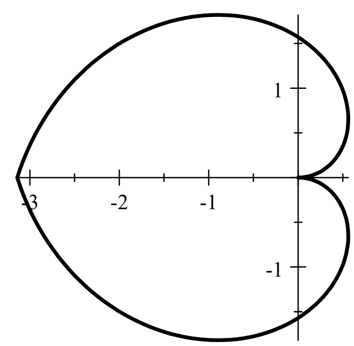
b. $-\sqrt{\pi}$

c. $1$

d. $\sqrt{\pi}$

e. $\pi$

4. Find the area of the heart shaped region inside the polar curve $r = |\theta|$.




a. $\frac{\pi^3}{6}$

b. $\frac{\pi^3}{3}$

c. $\frac{4\pi^3}{3}$

d. $\frac{8\pi^3}{3}$

e. $\frac{16\pi^3}{3}$

5. The solid half cylinder $0 \le y \le \sqrt{9-x^2}$ with $0 \le z \le 2$ has density $\delta = y$. Find the total mass.

a. $9$

b. $18$

c. $36$

d. $9\pi$

e. $18\pi$

**6**. The solid half cylinder $0 \le y \le \sqrt{9-x^2}$ with $0 \le z \le 2$ has density $\delta = y$.
Find the $y$-component of the center of mass.

**a**. $\frac{9\pi}{16}$
**b**. $\frac{9\pi}{4}$
**c**. $9\pi$
**d**. $\frac{81\pi}{4}$
**e**. $\frac{\pi}{2}$

**7**. Compute $\iiint \nabla \cdot \vec{F}\, dV$ on the solid hemisphere $0 \le z \le \sqrt{25-x^2-y^2}$
for the vector field $\vec{F} = \left(\frac{2}{3}x^3, \frac{2}{3}y^3, z(x^2+y^2+z^2)\right)$.

**a**. $0$
**b**. $5^3\pi^2$
**c**. $\frac{14}{3}\pi 5^4$
**d**. $6\pi 5^4$
**e**. $12\pi 5^4$

**8**. If $\vec{F} = (-yz, xz, z^2)$, compute $\vec{F} \cdot \vec{\nabla} \times \vec{F}$.

**a**. $z^3$
**b**. $z^3 - xyz$
**c**. $2z^3$
**d**. $2z^3 - 2xyz$
**e**. $0$

Work Out: (20 points each. Part credit possible. Show all work.)

**9**. (30 points) Consider the hemispherical surface

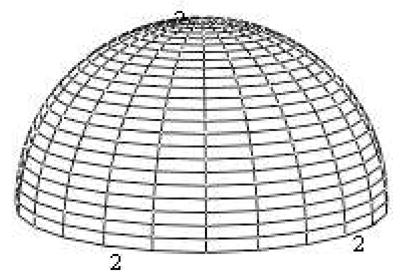
$$z = \sqrt{4 - x^2 - y^2}$$

which may be parametrized by

$$\vec{R}(\varphi,\theta) = (2\sin\varphi\cos\theta, 2\sin\varphi\sin\theta, 2\cos\varphi)$$

and the vector field $\vec{F} = (-y^3, x^3, z(x^2 + y^2))$.

Find each of the following:




**a**. (2 pts) $\vec{e}_\varphi =$

**b**. (2 pts) $\vec{e}_\theta =$

**c**. (3 pts) $\vec{N} =$

**d**. (2 pts) $|\vec{N}| =$

**e**. (5 pts) The total mass of the surface if the surface density is $\delta = z$.

$M =$

**f**. (6 pts) The $z$-component of the center of mass of the surface if the surface density is $\delta = z$.

$M_z =$

$\bar{z} =$

**g**. (3 pts) $\nabla \times \vec{F} =$

(continued)

**h**. $(2\text{ pts})$ $\nabla \times \vec{F}\left(\vec{R}(\varphi,\theta)\right) =$

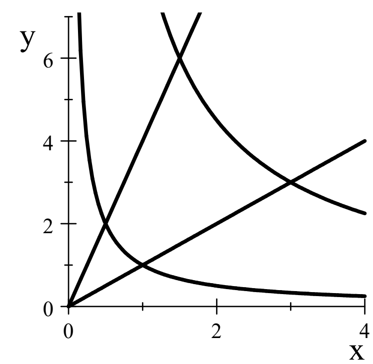
**i**. $(5\text{ pts})$ $\iint \nabla \times \vec{F} \bullet d\vec{S}$ with normal pointing up.

**10**. $(16\text{ points})$ Compute $\displaystyle\iint_R y^2\, dx\,dy$ over the diamond shaped region $R$ bounded by

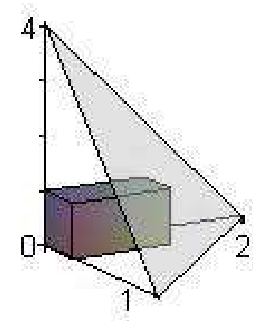
$$y = \frac{1}{x}, \qquad y = \frac{9}{x}, \qquad y = x, \qquad y = 4x$$

FULL CREDIT for integrating in the curvilinear coordinates $(u,v)$ where $u^2 = xy$ and $v^2 = \dfrac{y}{x}$.

HALF CREDIT for integrating in rectangular coordinates.



**11**. (15 points) Find the volume of the largest rectangular box in the first octant with three faces in the coordinate planes and one vertex on the plane $x + \frac{y}{2} + \frac{z}{4} = 1$.




Solve either by Eliminating a Variable or by Lagrange Multipliers.
**5 points extra credit** for doing both. Clearly separate solutions.