1-11	/55
12	/10
13	/30
14	/12
Total	/107

Name________________________

MATH 253 Final Exam Fall 2016

Sections 201/202 P. Yasskin

Multiple Choice: (5 points each.)

Two questions have part credit, but I won't say which.

1. Find the tangential acceleration along the curve $\vec{r}(t) = (3t^2, 4t^3, 3t^4)$.

a. $6 - 36t^2$

b. $6 + 36t^2$

c. $6 - 24t^2$

d. $6 + 24t^2$

e. $12\sqrt{3 + 4t^2 + 9t^4}$

2. The graphs of $z = x^2 + y^3$ and $z = 129x^2 - y^3$ intersect in the curve $\vec{r}(t) = (x(t), y(t), z(t))$.
If $x(t) = t^3$, then $z(t) =$

a. $63t^6$

b. $64t^6$

c. $65t^6$

d. $127t^6$

e. $128t^6$

3. Antwoman is currently running across a frying pan. She is currently at the point $(3,2)$ and has speed 20 cm/sec in the direction $\left(\frac{3}{5}, \frac{4}{5}\right)$. She measures the temperature to be $325°$K and its gradient to be $(-5,2)$ °K/cm. At what rate (in °K/sec) does she see the temperature changing?

a. -28

b. -14

c. -1.4

d. 1.4

e. 28

4. Find 3 positive numbers x, y and z, whose sum is 90 such that $f(x,y,z) = xy^2z^3$ is a maximum. What is xyz?

a. $2^3 \cdot 3 \cdot 5^3 \cdot 7$

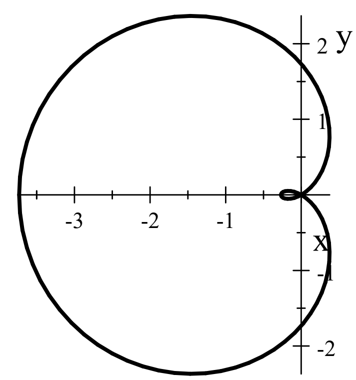
b. $2^2 \cdot 3^2 \cdot 5^4$

c. $2 \cdot 3^4 \cdot 5^3$

d. $2^2 \cdot 5^3 \cdot 7^2$

e. $2 \cdot 3 \cdot 5^4 \cdot 7$

5. Find the area inside the **inner** loop of the limacon $r = \sqrt{3} - 2\cos\theta$.



a. $\frac{5}{3}\pi + \frac{1}{2}\sqrt{3}$

b. $\frac{5}{3}\pi + \frac{1}{2}\sqrt{3} - 6$

c. $6 - \frac{1}{2}\sqrt{3} - \frac{5}{3}\pi$

d. $\frac{5}{6}\pi - \frac{3}{2}\sqrt{3}$

e. $\frac{3}{2}\sqrt{3} - \frac{5}{6}\pi$

6. Find the mass of the plate between $y = 2x$ and $y = x^2$ if the surface density is $\delta = xy$.

a. $\frac{8}{15}$
b. $\frac{16}{15}$
c. $\frac{32}{15}$
d. $\frac{8}{3}$
e. $\frac{16}{3}$

7. Find the y-component of the center of mass of the plate between $y = 2x$ and $y = x^2$ if the surface density is $\delta = xy$.

a. $\frac{8}{15}$
b. $\frac{5}{12}$
c. $\frac{12}{5}$
d. $\frac{15}{8}$
e. $\frac{32}{5}$

8. Compute $\int_{(0,0,0)}^{(1,2,0)} \vec{F} \cdot d\vec{s}$ for $\vec{F} = (2x + y, x + 2y, 2z)$ along the curve $\vec{r}(t) = (\sqrt{t}, t^2 + t^3, t^2 - t^3)$.
HINT: Find a scalar potential if possible.

a. 1
b. 3
c. 5
d. 7
e. Cannot be computed because there is no scalar potential.

9. Compute $\int_{(0,0,0)}^{(1,2,0)} \vec{F} \cdot d\vec{s}$ for $\vec{F} = (x,x,x)$ along the curve $\vec{r}(t) = (t, t^2 + t^3, t^2 - t^3)$.

HINT: Find a scalar potential if possible.

a. $\frac{11}{6}$

b. $-\frac{5}{6}$

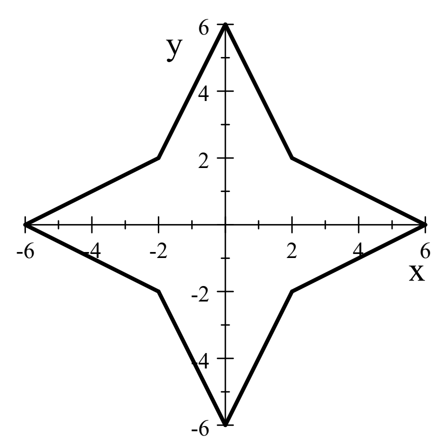
c. $\frac{5}{6}$

d. 3

e. Cannot be computed because there is no scalar potential.

10. Compute the line integral $\oint \vec{F} \cdot d\vec{s}$ for $\vec{F} = (2y - 3xy^2, 3x - 3x^2y)$ counterclockwise around the boundary of the region shown at the right.

HINT: Use a Theorem.



a. 3

b. 6

c. 12

d. 24

e. 48

11. Compute $\iint_Q \vec{\nabla} \times \vec{F} \cdot d\vec{S}$ for

$\vec{F} = \left(x(z-16)^2 - yz^3, y(z-16)^2 + xz^3, x^4 z - y^4 z\right)$

over the quartic surface Q given by $z = (x^2+y^2)^2$

for $z \le 16$ oriented down and out, which may be

parametrized by $\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, r^4)$.

HINT: Use a Theorem. Be sure to check the orientation.

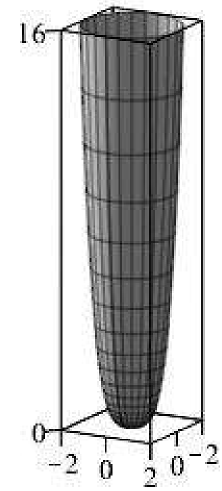
a. $-2^{18}\pi$

b. $-2^{16}\pi$

c. $-2^{15}\pi$

d. $2^{15}\pi$

e. $2^{18}\pi$



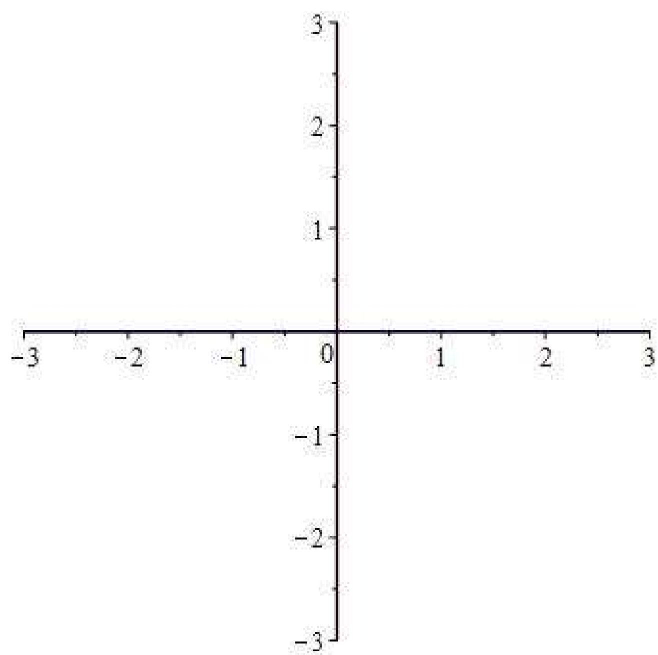
Work Out: (Points indicated. Part credit possible. Show all work.)

12. (10 points) Roughly draw the contour plot for the function $f(x,y) = x^2 y$.

Include and label the level sets for

$f = -2, -1, 0, 1, 2.$

If there is more than one piece to a level set, label each piece.

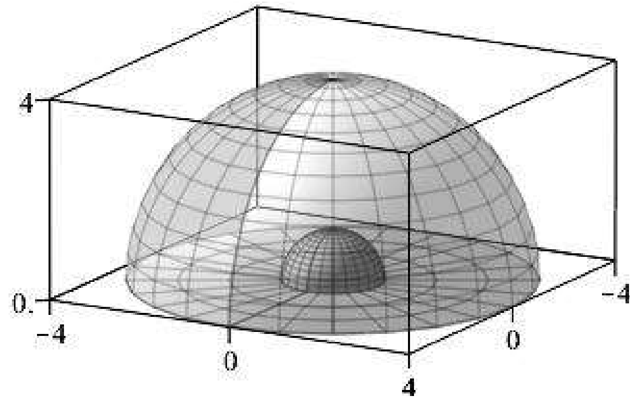


13. (30 points) Verify Gauss' Theorem

$$\iiint_V \vec{\nabla} \cdot \vec{F}\, dV = \iint_{\partial V} \vec{F} \cdot d\vec{S}$$

for the vector field $\vec{F} = (xz^2, yz^2, z^3)$ and the solid region, V, between the hemispheres $z = \sqrt{16 - x^2 - y^2}$ and $z = \sqrt{1 - x^2 - y^2}$ for $z \ge 0$.

Use the following steps: Be sure to check orientations.



a. (7 pts) LHS:

$\vec{\nabla} \cdot \vec{F} =$

Name your coordinate system ______________________ and evaluate

$\vec{\nabla} \cdot \vec{F}\Big|_{\vec{R}} =$ $dV =$

$\iiint_V \vec{\nabla} \cdot \vec{F}\, dV =$

b. RHS: The boundary consists of 3 pieces.

i. (13 pts) Parametrize the Outer Hemisphere: $\vec{R}(\varphi, \theta) =$

Evaluate the vector field on the surface:

$\vec{F}\left(\vec{R}\right) = (xz^2, yz^2, z^3) =$

Find the normal

$\vec{e}_\varphi =$

$\vec{e}_\theta =$

$\vec{N} =$

Evaluate:

$\vec{F} \cdot \vec{N} =$

$\iint_{\text{outer}} \vec{F} \cdot d\vec{S} =$ (continued)

ii. (3 pts) Parametrize the Inner Hemisphere: $\vec{R}(\varphi,\theta) =$

Evaluate the vector field on the surface:

$\vec{F}\left(\vec{R}\right) = (xz^2, yz^2, z^3) =$

Find the normal:

$\vec{e}_\varphi =$

$\vec{e}_\theta =$

$\vec{N} =$

Evaluate:

$\vec{F} \cdot \vec{N} =$

$\iint_{\text{inner}} \vec{F} \cdot d\vec{S} =$

iii. (3 pts) Parametrize the Base Ring: $\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, 0)$

Evaluate the vector field on the surface:

$\vec{F}\left(\vec{R}\right) = (xz^2, yz^2, z^3) =$

Find the normal:

$\vec{e}_r =$ $\vec{e}_\theta =$ $\vec{N} =$

Evaluate:

$\iint_{\text{ring}} \vec{F} \cdot d\vec{S} =$

iv. (2 pts) Total RHS

$\iint_{\partial V} \vec{F} \cdot d\vec{S} =$

c. (2 pts) Comparison of LHS and RHS:

14. (12 points) Find all points on the paraboloid $z = x^2 + y^2$ where the normal line passes throught the point $P = (0,0,36)$.

HINT: The normal vector at $X = (x,y,z)$ must be parallel to the vector $\overrightarrow{XP}$.