

Test 1

Problem 1 (25 pts.) Let ℓ_0 be the line in \mathbb{R}^3 passing through the point $\mathbf{a} = (1, 1, 0)$ in the direction $\mathbf{v} = (1, 1, 1)$. Let Π be the plane in \mathbb{R}^3 passing through the line ℓ_0 and the point $\mathbf{b} = (0, 1, 1)$. Let ℓ be the line in \mathbb{R}^3 passing through the points $\mathbf{c} = (1, 0, 1)$ and $\mathbf{d} = (2, 0, 2)$.

- (i) Find a parametric representation for the line ℓ .
- (ii) Find a parametric representation for the plane Π .
- (iii) Find an equation for the plane Π .
- (iv) Find the point where the line ℓ intersects the plane Π .
- (v) Find the angle between the line ℓ and the plane Π .
- (vi) Find the distance from the point $(1, 1, 1)$ to the plane Π .

Problem 2 (15 pts.) Find a quadratic polynomial $p(x)$ such that $p(-1) = 1$, $p(2) = -2$, and $p(3) = 1$.

Problem 3 (20 pts.) Let $A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & -1 & 2 & 1 \end{pmatrix}$. Find the inverse matrix A^{-1} .

Problem 4 (20 pts.) Let A be the same matrix as in Problem 3. Evaluate the following determinants:

- (i) $\det A$;
- (ii) $\det(A - I)$;
- (iii) $\det(2A)$.

Bonus Problem 5 (20 pts.) Let $\mathbf{v}_1 = (1, 1, 0)$, $\mathbf{v}_2 = (0, 1, 1)$, $\mathbf{v}_3 = (1, 1, 1)$, and $\mathbf{v}_4 = (0, 1, 0)$. Determine which of the following sets of vectors are linearly independent:

- (i) $\mathbf{v}_1, \mathbf{v}_2$;
- (ii) $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$;
- (iii) $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$.