MATH 323 Fall 2023

# Homework assignment #4

**Problem 1.** Let $\mathbf{0}$ be the zero vector of a vector space $V$. Prove that $\beta\mathbf{0} = \mathbf{0}$ for each scalar $\beta$.

**Problem 2.** Consider the set $V = \mathbb{R}^2$ with addition $\oplus$ and scalar multiplication $\odot$ defined by

$$(x_1, x_2) \oplus (y_1, y_2) = (x_1 + x_2, y_1 + y_2),$$
$$\alpha \odot (x_1, x_2) = (\alpha x_1, x_2).$$

Is $V$ a vector space with these operations? Justify your answer.

**Problem 3.** Determine whether the following sets form subspaces of the vector space $\mathbb{R}^2$.
**(i)** The set $S_1$ of all vectors $(x_1, x_2) \in \mathbb{R}^2$ such that $x_1 x_2 = 0$.
**(ii)** The set $S_2$ of all vectors $(x_1, x_2) \in \mathbb{R}^2$ such that $x_1 = 3x_2$.

**Problem 4.** Determine whether the following vectors form a spanning set for the vector space $\mathbb{R}^2$:

**(i)** $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 4 \\ 6 \end{pmatrix}$; **(ii)** $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 4 \end{pmatrix}$.

**Problem 5.** Let $\mathbf{v} = (-1, 2, 3)$ and $\mathbf{w} = (3, 4, 2)$. Determine whether the following vectors belong to $\mathrm{Span}(\mathbf{v}, \mathbf{w})$, the span of $\mathbf{v}$ and $\mathbf{w}$:

**(i)** $(2, 6, 6)$, **(ii)** $(-9, -2, 5)$.

**Problem 6.** Determine whether $\{x + 2, x + 1, x^2 - 1\}$ is a spanning set for $\mathcal{P}_3$, the vector space of polynomials of degree at most 2.

**Problem 7.** Suppose that $U_1$ and $U_2$ are subspaces of the same vector space $V$. Prove that their intersection $U_1 \cap U_2$ is also a subspace of $V$.