

**Homework assignment #8**

**Problem 1 (2 pts).** A *Gaussian integer* is a complex number of the form  $m + in$ , where  $m, n \in \mathbb{Z}$ .

- (i) Show that the Gaussian integers form an integral domain.
- (ii) Which Gaussian integers have a multiplicative inverse?
- (iii) Describe the field of quotients of the Gaussian integers in  $\mathbb{C}$ .

**Problem 2.** Let  $D$  be an integral domain. Prove that 1 and  $-1$  are the only elements of  $D$  that are their own multiplicative inverse.

**Problem 3.** Let  $\mathbb{F}$  be a finite field. Prove that the product of all nonzero elements of  $\mathbb{F}$  equals  $-1$ . [Hint: use the previous problem.]

**Problem 4.** Prove that for any prime number  $p$ , the number  $(p-1)! + 1$  is divisible by  $p$ . [Hint: use the previous problem.]

**Problem 5 (2 pts).** For any integer  $n \geq 2$ , determine the remainder after division of the number  $(n-1)!$  by  $n$ .

**Problem 6.** Determine the remainder after division of the number  $2021^{2022^{2023}}$  by 11.

**Problem 7.** Find all integer solutions of the equation  $15x - 23y = 40$ .

**Problem 8.** Find all integer solutions of the equation  $13x + 31y = 3$ .