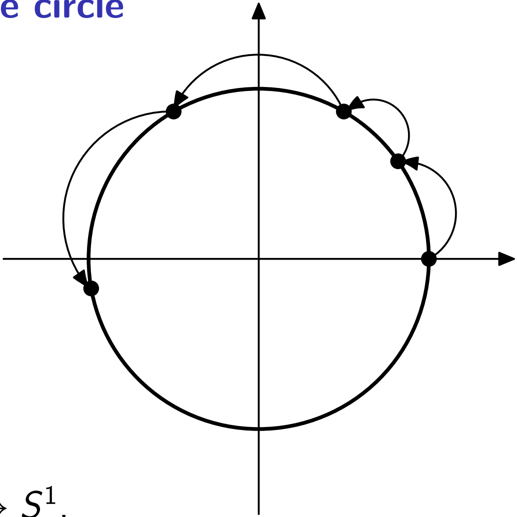
MATH 614

Dynamical Systems and Chaos

Lecture 17a: Morse-Smale diffeomorphisms.

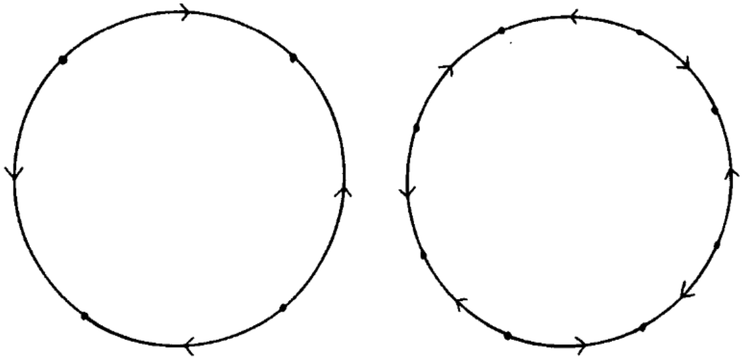
Maps of the circle



$T : S^1 \to S^1$,

T an orientation-preserving homeomorphism.

Structurally stable maps of the circle



Definition. An orientation-preserving diffeomorphism $f : S^1 \to S^1$ is **Morse-Smale** if it has rational rotation number and all of its periodic points are hyperbolic.

If $\rho(f) = m/n$, a reduced fraction, then all periodic points of f have period n. Hence the only periodic points of f^n are fixed points, alternately sinks and sources around the circle.

Theorem A Morse-Smale diffeomorphism of the circle is C^1-structurally stable.

Theorem (The Closing Lemma) Suppose f is a C^r-diffeomorphism of S^1 with an irrational rotation number. Then for any $\varepsilon > 0$ there exists a diffeomorphism $g : S^1 \to S^1$ with a rational rotation number such that f and g are C^r-ε close.

Theorem (Kupka-Smale) For any orientation-preserving C^r-diffeomorphism f of S^1 and any $\varepsilon > 0$ there exists a Morse-Smale diffeomorphism that is C^r-ε close to f.