Homework Assignment 10 in Differential Equations, MATH308-Fall 2016

due November 14, 2016

Topics covered : *The Phase Plane: linear systems (section 9.1): types of critical (equillibrium) points and stability; sketch of the Locally linear systems (section 9.3)*

1. For each of the following systems

   i) Find and *classify* the critical (equilibrium) point and determine whether it is stable, asymptotically stable, or unstable;

   ii) *Sketch* the phase portrait of the system (indicating *direction of motion* along trajectories by arrows and the direction of eigenvectors, if relevant):

   (a)
$$\begin{cases} x_1' &= -5x_1 + 15 \\ x_2' &= -5x_2 - 10, \end{cases}$$

   (b)
$$\begin{cases} x_1' &= -9x_1 + 6x_2 + 3 \\ x_2' &= 4x_1 + x_2 - 16, \end{cases}$$

   (c)
$$\begin{cases} x_1' &= 3x_1 - 4x_2 \\ x_2' &= 5x_1 + 7x_2 - 41, \end{cases}$$

   (d)
$$\begin{cases} x_1' &= 6x_1 - 13x_2 + 1 \\ x_2' &= x_1 - 8x_2 + 6, \end{cases}$$

   (e)
$$\begin{cases} x_1' &= -10x_1 + 6x_2 + 2 \\ x_2' &= -6x_1 + 2x_2 + 14. \end{cases}$$

2. For the following system
$$\begin{cases} x' &= x(16 - 3x - 2y) \\ y' &= y(26 - 4y - 5x) \end{cases} \tag{1}$$

   (a) Determine all critical points.

   (b) Find the corresponding linear system near each critical point.

   (c) Based on this linear systems determine the type of each critical point and their stability properties (i.e. whether they are stable, asymptotically stable, or unstable)?

   (d) Sketch the phase portrait of system (1) in the first quadrant.

   (e) **bonus-10 points** System (1) corresponds to a model of competing species. Review the end of class notes of Wednesday, November 9, regarding an example of a model of competing species and section 9.4 in the textbook. Based on your analysis in the previous item, answer the following question: does the coexistence occurs in the model given by system (1)?