Homework Assignment 16 in Differential Equations, MATH308-SPRING 2015

due May 1, 2015

Sections covered: 7.9 (the method of variation of parametes for nonhomogeneous systems) and 7.8 (the case of repeated eigenvalues of (algebraic) multiplicity 3 *as a bonus)*

1. Use the method of variation of parameters to solve the following initial value problem:

$$\begin{cases} x_1' = 3x_1 - x_2 + 4e^{2t} \\ x_2' = -x_1 + 3x_2 + 4e^{4t} \\ x_1(0) = 1, x_2(0) = 1. \end{cases}$$

2. Use the method of variation of parameters to find the general solution of the following system:

$$X' = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} X + \begin{pmatrix} \cos t \\ \sin t \end{pmatrix} e^t.$$

3. (**bonus 80 points**) Find the general solution of the following systems:

   (a)
$$X' = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 2 & -1 \\ 0 & 1 & 0 \end{pmatrix} X;$$

   (b)
$$\begin{cases} x_1' &= & -9x_1 + x_2 - 2x_3 \\ x_2' &= & x_1 - 9x_2 + 2x_3 \\ x_3' &= & x_1 - x_2 - 6x_3, \end{cases}$$

   if it is known that $\lambda = -8$ is an eigenvalue of algebraic multiplicity 3.