Darboux frame. Assume that k_1 and k_2 are principal curvatures on an oriented surface S and e_3 is the fid of normal vectors to S. Assume that p_0 is a non-umbilical point on S and e_1 and e_2 are two unit vector fields in a neighborhood U of p_0 in S such that: they generate the principal directions corresponding to the principal curvatures k_1 and k_2 respectively and they constitute a positive frame of the tangent space at any point of U (the frame e_1, e_2, e_3) is called the *Darboux frame*). Let $F : U \to ASO(3)$ be defined as follows: $x \in U \mapsto \big(x, e_1(x), e_2(x), e_3(x)\big)$. Assume also that χ_1 and χ_2 are geodesic curvatures of the lines of curvatures tangent to e_1 and e_2 respectively.

a) Calculate $F^*(\omega_1^2)$ in terms of χ_1 and χ_2 , where ω_1^2 is the corresponding entry of the Maurer-Cartan form of $AS0(3)$.

b) Prove that $[e_1, e_2] = -\chi_1 e_1 + \chi_2 e_2$.

c) Prove that the the functions k_1, k_2, χ_1, and χ_2 satisfies the following 3 relations (which are exactlyof the Gauss and Codazzi equations in the Darboux frame):

$$
\begin{aligned}
&k_1 k_2 = e_1(\chi_2) + e_2(\chi_1) - (\chi_1^2 + \chi_2^2),\\
&e_1(k_2) = \chi_2(k_2 - k_1),\\
&e_2(k_1) = \chi_1(k_1 - k_2).
\end{aligned}
$$