## Course Information

Course Number: MATH 622
Course Title: Differential Geometry I
Section: 600
Time: TR 09:35 am-10:50 am
Location: Bloc 110
Credit Hours: 3

## Instructor Details

Instructor: **Igor Zelenko**
Office: 601J
Phone: 979-820-0620
E-Mail: zelenko@math.tamu.edu
Office Hours: Monday 10:30 am-noon via Zoom (the link to Zoom room is provided on Canvas)
Course Webpage: (https://canvas.tamu.edu/courses/117201)

## Textbook and/or Resource Materials

**Main Texts:**

**Liviu Nicolaescu**, *Lectures on Geometry of Manifolds*, 2nd Edition, World Scientific Publishing, 2007

Supplementary Texts (we give reference to relevant items of Course description for which the book will be used):

1. **Frank Warner,** Foundations of Differentiable Manifolds and Lie Groups, Springer Graduate Texts in Mathematics, v.94

*This book is very concise and contains in particular a very thorough treatment of the most material of the first three chapters of the book (and more) but it is too dry and abstract and will be used as the source for additional exercises and for references.*

2**. Manfredo P. Do Carmo**, *Differential Geometry of Curves and Surfaces*: Revised and updated Second Edition (Dover Books on Mathematics).

*This book as the reference for elementary differemtial geometry of the undergraduate level that we will review in item 2 (see the course description)*

3. **R.W. Sharpe,** Differential Geometry: Cartan's Generalization of Klein's Erlangen Program, Springer Graduate Texts in Mathematics, Vol. 166.

*This text will be used for treatment of item 4 of the course description: Frobenius theorem (chapter 2 there) and of Maurer -Cartan form on the Lie group (Chapter 3 there) as the basis for Cartan's method of moving frame ( paragraph 4.2.3 of the main text).*

4. **M. Do Carmo**, *Differential Forms and Applications (Universitext).*

*This is an elementary and self-contained treatment of differential forms, manifold, generalized Stokes theorem, and applications to intrinsic and extrinsic geometry of surfaces. It might be useful in addition to the main text if you feel that the treatment of the same subjects in the main text is too abstract (and especially for the Gauss -Bonnet theorem., item 7 of the course description.*

5. **John Lee**, *Introduction to Smooth Manifolds* , Graduate Texts in Mathematics, Vol. 218),2nd Edition

*This text will be used as the source for additional exercises and for references. It is a very detailed and more elementary treatment of most of the first three chapters of the main text (except the section 3.3). It is recommended for those who like to see all details.*

6. **Michael Spivak** , *A Comprehensive Introduction to Differential Geometry,* ***vol.1***

*This is the first volume of the famous classical 5 volumes series and it seems that the book of Lee was written based on this classical book. It will be used for references.You may enjoy reading this classics.*

7 **Michael Spivak** , *A Comprehensive Introduction to Differential Geometry,* ***vol.2***

*This is the second volume of the famous classical 5 volumes series. It roughly contains the material of Section 3.3 and Chapter 4 of the main book. The most valuable feature that the theory is given according to its historical development , from Gauss, Riemann, and Ricci to Cartan and Kozhul (including the translation of the original Gauss's and Riemann's works) . It will be used for references.You may enjoy reading this classics.*

**8. Walter Rudin,** *Principles of Mathematical Analysis, chapters 9 and 10 there*

*This text provides the analytic background for the course. In particular, chapter 9 contains all waht you need to know about differentiable function, contraction principles and proofs of the Inverse and Implicit function Theorems (that are only sketched in the main text). Chapter 10 contains the material on integration of differential forms and generalized Stockes' theorem.*

**Course Prerequisites**

**Prerequisite:** Some basic concepts from Linear algebra and Topology.

The course does not have any graduate course as a prerequisite. The undergraduate prerequisites are standard Calculus courses (MATH 171/151, 172/152, 221/251 or equivalent), Differential Equations (MATH 308), and Linear algebra (MATH 304 or 323). I will give all mathematical background beyond the above courses in the class.

## Course Description

This is the first semester of a year-long graduate course in differential geometry. We will discuss the following topics this semester:

1. Manifolds and submanifold via relevant examples needed in the future such as classical Lie groups (and their Lie algebras), projective spaces and Grassmanians (and their tangent spaces).
2. Classical differential geometry of curves in R^3 (and also in R^n): the Frenet frame, curvature, torsion ( to use it as the motivating example for the notions of Lie groups, Lie algebras, homogeneous spaces, and Maurer-Cartan form). Basics of classical differential geometry of surfaces (without differential forms): the Gauss map, the first and second fundamental forms, the Gaussian and mean curvatures (the rest of this classical theory together with a revision of the same notions will be given later, in item 6, as application of differential forms).
3. Basic concepts of tensors and exterior differential calculus: tensor algebra, exterior algebra, vector fields and flows, Lie brackets of vector fields, tensor fields, Lie derivatives of tensor fields, differential forms and exterior differential of them, the notions of pull-back and push-forward.
4. Stokes' Theorem (including manifolds with boundary, orientability and orientations).
5. Distributions, foliations, integral submanifolds, Frobenius theorem.
6. Homogeneous spaces, submanifolds in homogeneous spaces: the minimal goal of this item is to revise and continue the topic of item 2, classical (local) differential geometry of surfaces in R^3 via differential forms: Cartan's lemma, second fundamental form in this language, the Gauss Egregium theorem, the Gaussian curvature of two-dimensional Riemannian manifold, the Codazzi equation, and the fundamental theorem of surface theory (the Bonnet theorem). However, this topic will be given from more general prospective of geometry of submanifolds in homogeneous spaces as an important application of the Frobenius theorem via the Maurer-Cartan form on a Lie group, the notion of the Darboux derivative of a map from a manifold to a Lie group and the fundamental theorem for Darboux derivatives via the Cartan's graph method
7. The Gauss-Bonnet theorem.
8. The concepts of vector bundles, connections on vector bundles, the curvature tensor of a connection, the curvature form of a connection in local trivialization, parallel transport and geodesics, Riemannian manifolds and Levi-Civita connection.

## Grading Policy

Your final grade will be determined by your performance on the homework, the middleterm exam, and the final exam. The grade ingredients are:

| Activity | % |
|---|---|
| **Weekly Homework Assignments** | **40%**: It will be usually posted on Tuesdays with due date next Tuesday Homework submission will be via Canvas |
| **One midterm exam** | **30%:** The midterm exam will take place in some evening, out of regular class. The midterm will be in the first week after the Spring break (i/e/, the week of March 21-25). The exact dates/time/location of the midterm will be announced well in advance. |
| **Final exam** | 30%: the final will be **on May 5 12:30 – 2:30 p.m.** in the regular classroom. |
| **Total** | 100% |

**Grading Scale**

| Range | Grade |
|---|---|
| **90 -100 %** | A |
| **80- 89 %** | B |
| **70-79 %** | C |
| **60-69 %** | D |
| **0-59 %** | F |

**TECHNOLOGY**: You will need to scan and upload some of your written work as a PDF (this can be achieved with a cell phone or other technology). Course notes and other materials will be posted on Canvas.
If the university moves to on-line instruction, then exams will be proctored on-line over Zoom. To do this, the following technical requirements are needed:

- Appropriate hardware (laptop or desktop computer, a second device such as a mobile phone, high-speed internet connection)
- Appropriate software (PDF reader, Zoom on phone and computer, the latest update on an internet browser-Chrome or Firefox are recommended)

## Grade complaints:

If you think a homework was graded incorrectly you have one week from the time the graded assignment was returned to you to bring the issue to the instructor's attention. No complaints after that time will be considered.

## Late Work Policy

Late work will NOT be accepted unless you have a University approved reason and contact me within two working days of the missed assignment.

## Appeal Policy

Students have one week upon the return of individual grades to notify the instructor of any inaccuracies in their graded work. Students should bring all grade disputes to their instructor in an individual Zoom meeting. Due to FERPA privacy issues, grade disputes will not be discussed over email or in the classroom.

## Working with Friends

Working together on homework is fine and encouraged, but each of you **must write up your own solutions in your own words, notation and/or symbols and write the names of your collaborators at the top left corner of your homework.** Copying a solution from a source and referencing the source is still considered a violation of academic integrity because you are submitting work for a grade that is not your own work. It is NOT permissible to discuss any aspect of any quiz, test or examination until ALL students have completed it. The penalties for violating this policy will range from an F on an assignment or test, to failing in the course.

## Attendance

Attendance is essential to complete this course successfully.
**Excused Absences:** University student rules concerning excused and unexcused absences, as well as makeups, can be found at http://student-rules.tamu.edu/rule07. Make-up exams and quizzes or late homework, writing assignments will NOT be allowed unless a University approved reason is given to me in writing. Notification before the absence is required when possible. Otherwise (e.g., accident, or emergency), you must notify me within two business days of the missed exam, quiz, or assignment to arrange a makeup.

*Other Important Dates:* Jan. 24 (last day to add or drop a course**),** March 14 - March 18 (Spring break), April 15 (Reading Day), April 19 (last day for all students to drop courses with no penalty (Q-drop)).

## *Zoom Etiquette*

*OFFICE HOUR ATTENDEES*
*When joining office hours via ZOOM, please join with your audio off. Everyone attending office hours will be joining one room, so if you would like to ask a question during office hours, please "raise your hand" and wait to be called on. If you need to speak to me privately, and have not made an individual appointment with me, please let me know through a private CHAT message and I will move you to a breakout room where we can talk one-on-one.*

## Class Announcements

Class announcements will be posted in Canvas and sent to your university e-mail account (Make sure to check your notification preferences to control how the course updates are sent.) It is your responsibility to check your account and the course page and get familiar with the announcements.

# University Policies

## Attendance Policy

The university views class attendance and participation as an individual student responsibility. Students are expected to attend class and to complete all assignments.

Please refer to Student Rule 7 in its entirety for information about excused absences, including definitions, and related documentation and timelines.

## Makeup Work Policy

Students will be excused from attending class on the day of a graded activity or when attendance contributes to a student's grade, for the reasons stated in Student Rule 7, or other reason deemed appropriate by the instructor.

Please refer to Student Rule 7 in its entirety for information about makeup work, including definitions, and related documentation and timelines.

Absences related to Title IX of the Education Amendments of 1972 may necessitate a period of more than 30 days for make-up work, and the timeframe for make-up work should be agreed upon by the student and instructor" (Student Rule 7, Section 7.4.1).

"The instructor is under no obligation to provide an opportunity for the student to make up work missed because of an unexcused absence" (Student Rule 7, Section 7.4.2).

Students who request an excused absence are expected to uphold the Aggie Honor Code and Student Conduct Code. (See Student Rule 24.)

## Academic Integrity Statement and Policy

"An Aggie does not lie, cheat or steal, or tolerate those who do."

"Texas A&M University students are responsible for authenticating all work submitted to an instructor. If asked, students must be able to produce proof that the item submitted is indeed the work of that student. Students must keep appropriate records at all times. The inability to authenticate one's work, should the

instructor request it, may be sufficient grounds to initiate an academic misconduct case" (Section 20.1.2.3, Student Rule 20).

You can learn more about the Aggie Honor System Office Rules and Procedures, academic integrity, and your rights and responsibilities at aggiehonor.tamu.edu.

## Americans with Disabilities Act (ADA) Policy

Texas A&M University is committed to providing equitable access to learning opportunities for all students. If you experience barriers to your education due to a disability or think you may have a disability, please contact Disability Resources in the Student Services Building or at (979) 845-1637 or visit disability.tamu.edu. Disabilities may include, but are not limited to attentional, learning, mental health, sensory, physical, or chronic health conditions. All students are encouraged to discuss their disability related needs with Disability Resources and their instructors as soon as possible.

## Title IX and Statement on Limits to Confidentiality

Texas A&M University is committed to fostering a learning environment that is safe and productive for all. University policies and federal and state laws prohibit gender-based discrimination and sexual harassment, including sexual assault, sexual exploitation, domestic violence, dating violence, and stalking.

With the exception of some medical and mental health providers, all university employees (including full and part-time faculty, staff, paid graduate assistants, student workers, etc.) are Mandatory Reporters and must report to the Title IX Office if the employee experiences, observes, or becomes aware of an incident that meets the following conditions (see University Rule 08.01.01.M1):

- The incident is reasonably believed to be discrimination or harassment.
- The incident is alleged to have been committed by or against a person who, at the time of the incident, was (1) a student enrolled at the University or (2) an employee of the University.

Mandatory Reporters must file a report regardless of how the information comes to their attention – including but not limited to face-to-face conversations, a written class assignment or paper, class discussion, email, text, or social media post. Although Mandatory Reporters must file a report, in most instances, you will be able to control how the report is handled, including whether or not to pursue a formal investigation. The University's goal is to make sure you are aware of the range of options available to you and to ensure access to the resources you need.

Students wishing to discuss concerns in a confidential setting are encouraged to make an appointment with Counseling and Psychological Services (CAPS).
Students can learn more about filing a report, accessing supportive resources, and navigating the Title IX investigation and resolution process on the University's Title IX webpage.

## Statement on Mental Health and Wellness

Texas A&M University recognizes that mental health and wellness are critical factors that influence a student's academic success and overall wellbeing. Students are encouraged to engage in proper self-care

by utilizing the resources and services available from Counseling & Psychological Services (CAPS). Students who need someone to talk to can call the TAMU Helpline (979-845-2700) from 4:00 p.m. to 8:00 a.m. weekdays and 24 hours on weekends. 24-hour emergency help is also available through the National Suicide Prevention Hotline (800-273-8255) or at suicidepreventionlifeline.org.

## Classroom Facial Mask

To help protect Aggieland and stop the spread of COVID-19, Texas A&M University urges students to be vaccinated and to wear masks in classrooms and all other academic facilities on campus, including labs. Doing so exemplifies the Aggie Core Values of respect, leadership, integrity, and selfless service by putting community concerns above individual preferences. COVID-19 vaccines and masking — regardless of vaccination status — have been shown to be safe and effective at reducing spread to others, infection, hospitalization, and death.